Wireless Communications
From 5G and WiFi 6 to Low Power IoT

Lecture 4: OFDM

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Yesterday’s Lecture Was

A. Too Difficult: I understood nothing.

B. Difficult: I understood something but not everything.

C. Moderate: I understood most stuff but I have questions

D. Easy: I understood everything really well.

Multipath in the Wireless Channel is problematic since it creates:

A. Inter-Symbol-Interference

B. Pathloss

C. Frequency Selective Fading

D. Additive White Gaussian Noise

ISI is considered negligible if the delayed symbols arriving longer paths interfere with less than < 1% of the symbol length.

If the longest path in the channel delays the symbol by 10ns, what is the maximum bandwidth for which we can ignore ISI?

A. 1 MHz

B. 10 MHz

C. 100 MHz

D. You can never ignore ISI

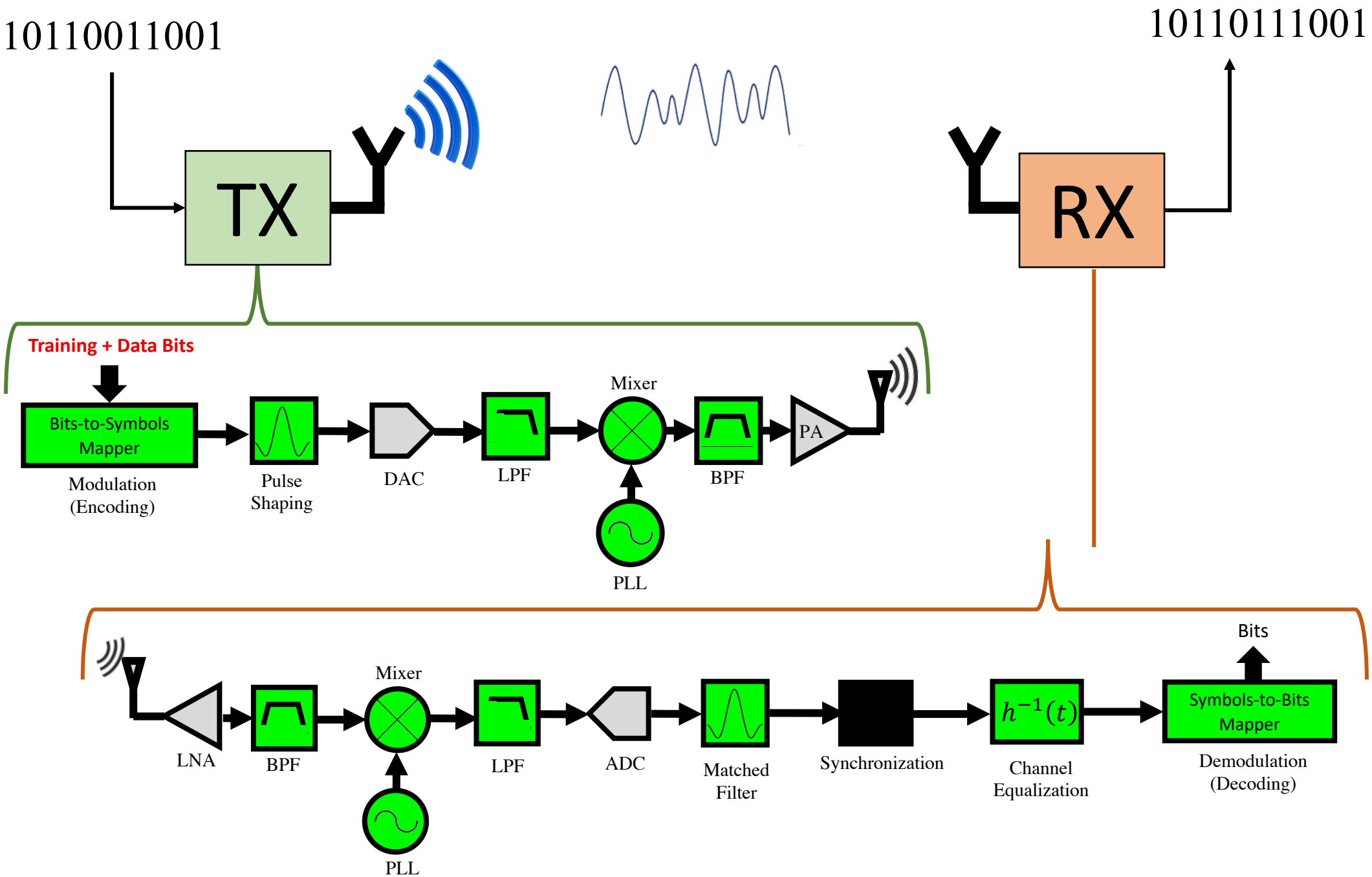
Previous Lecture:

- ✓ Pulse Shaping
- ✓ Matched Filter
- ✓ Multipath Channel
- ✓ Channel Estimation & Correction
- ✓ Narrowband vs. Wideband Channels
- ✓ Channel Equalization

This Lecture:

- ❑ Multi-Carrier Modulation
- ❑ Orthogonal Frequency Division Multiplexing (OFDM)
- ❑ OFDM Time Synchronization
- ❑ OFDM Frequency Synchronization
- ❑ OFDM Channel Estimation & Correction
- ❑ OFDM Phase Tracking

Wireless Communication System



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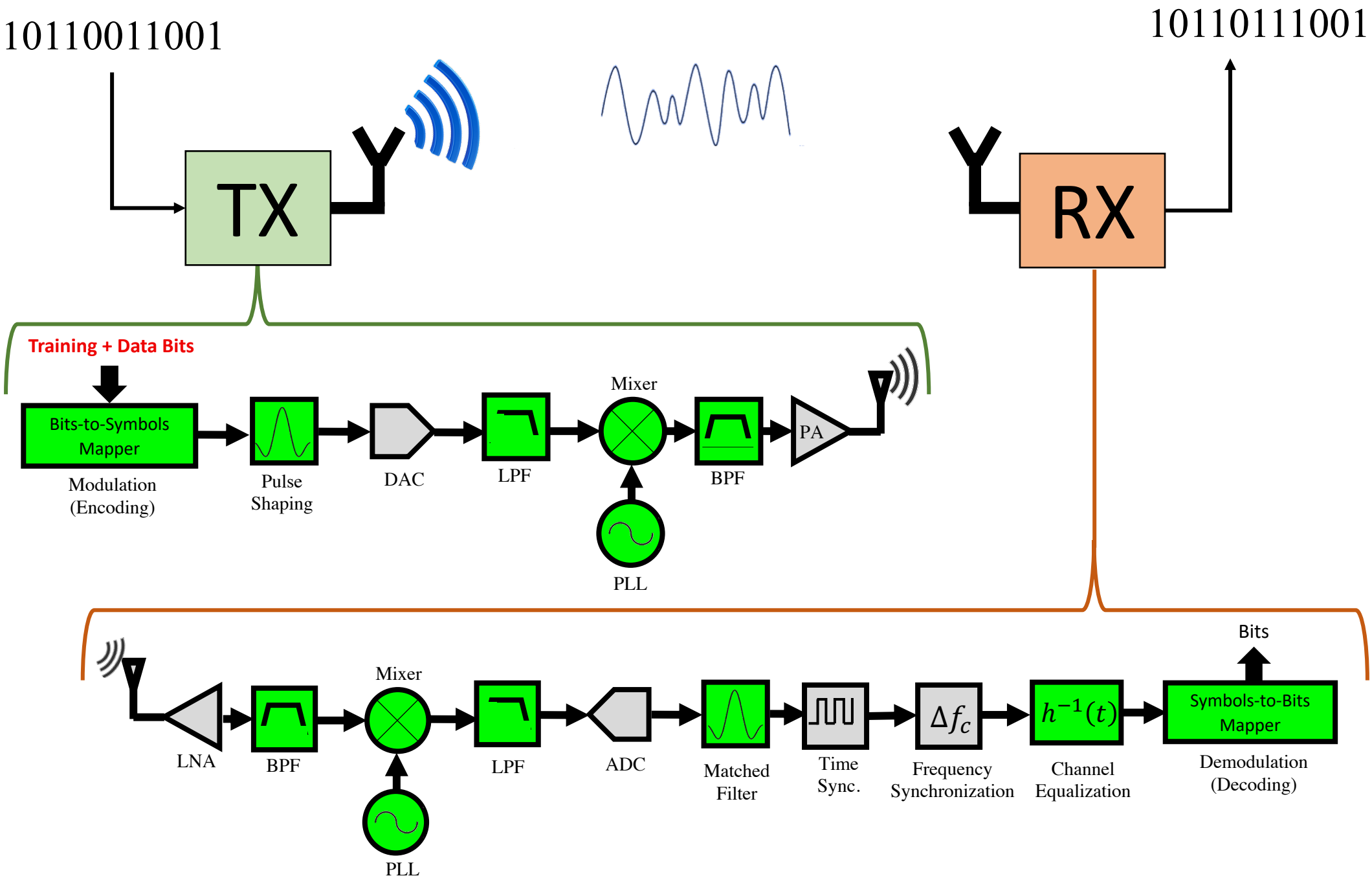

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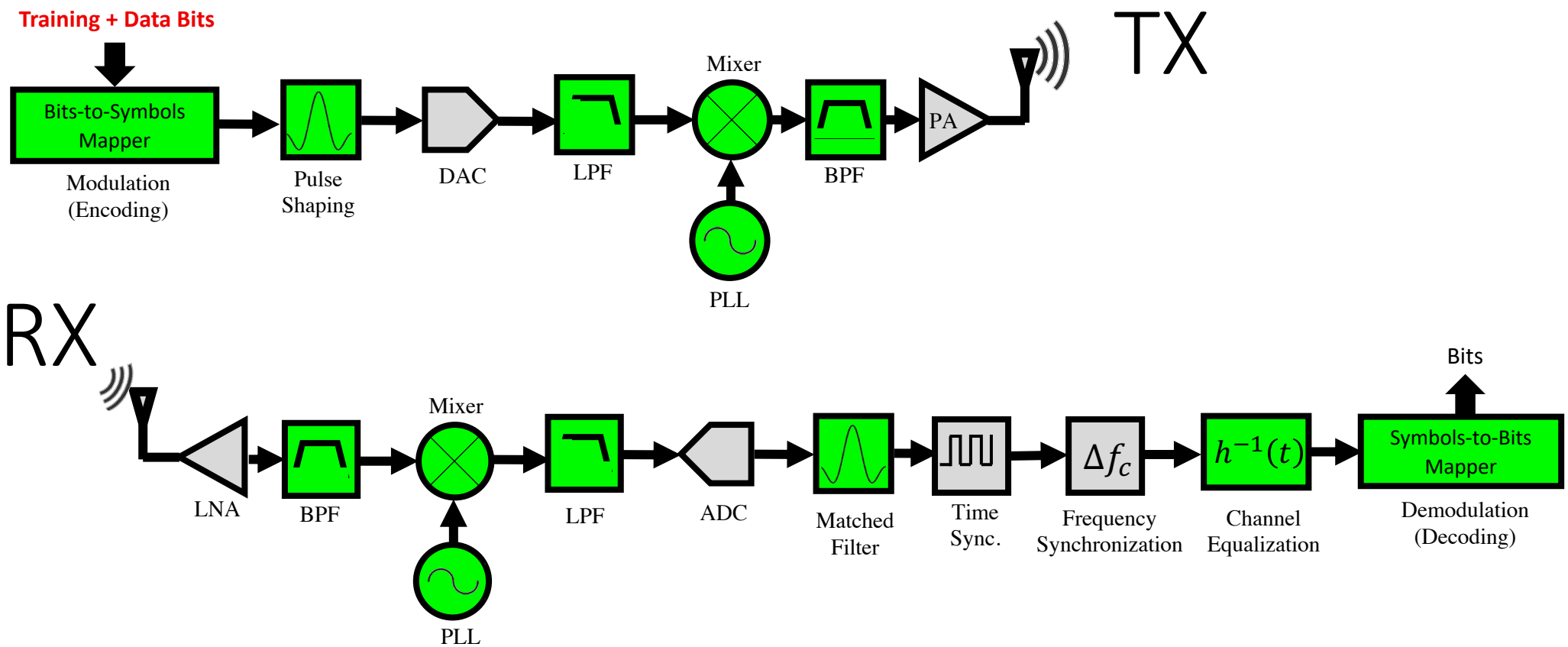


Single Carrier Modulation

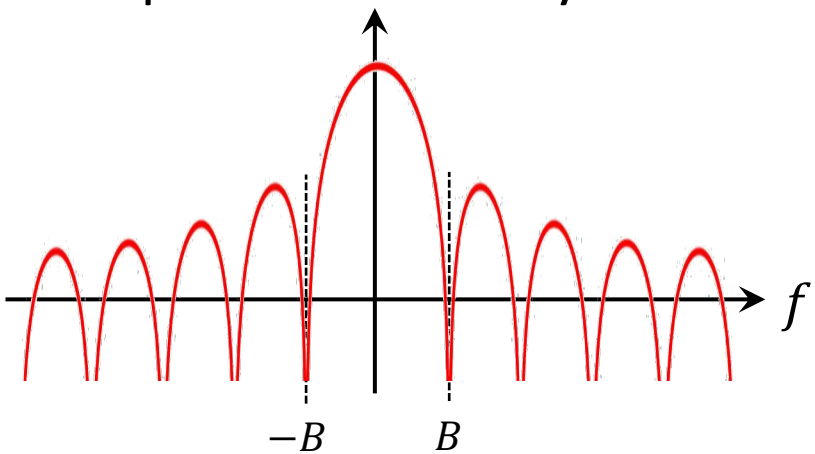
Symbols modulated on a single carrier frequency

$$s[n]\cos(2\pi f_c t)$$

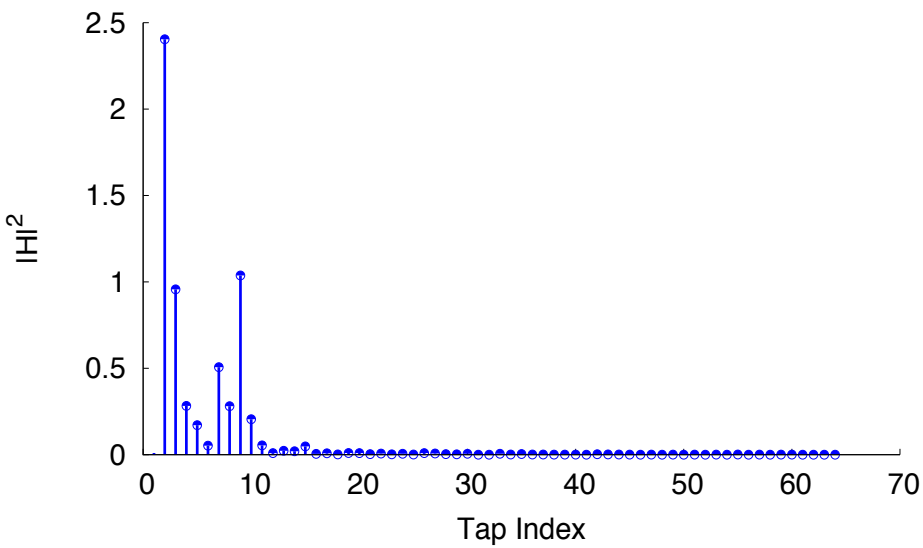
Single Carrier Modulation

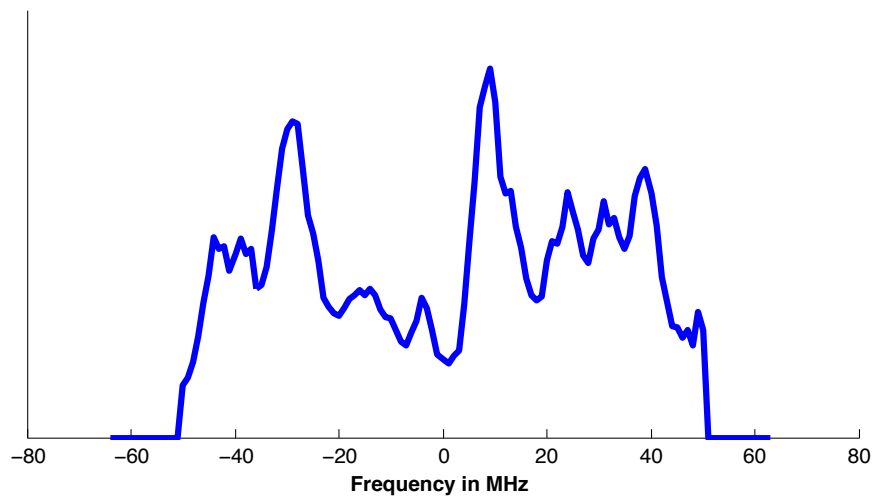
Symbols modulated on a single carrier frequency

- Low Spectral Efficiency: sinc & raised cosine leakage



- ISI: Inter-Symbol-Interference limits performance

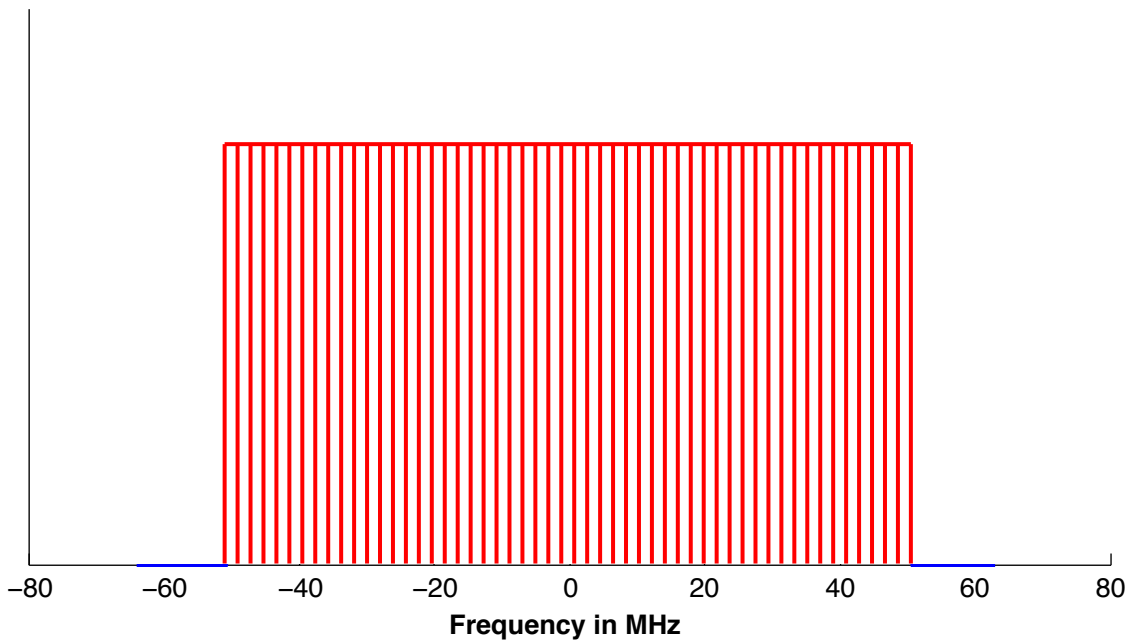




Multi-Carrier Modulation

Symbols modulated on multiple Sub-carrier frequencies

- Divide spectrum into many narrow bands



$$x(t) = \sum_i s_i[n] \mathrm{e}^{-j2\pi f_i t}$$

- Transmit symbols on different carriers in narrow bands
- Channel is Flat → No need to worry about ISI

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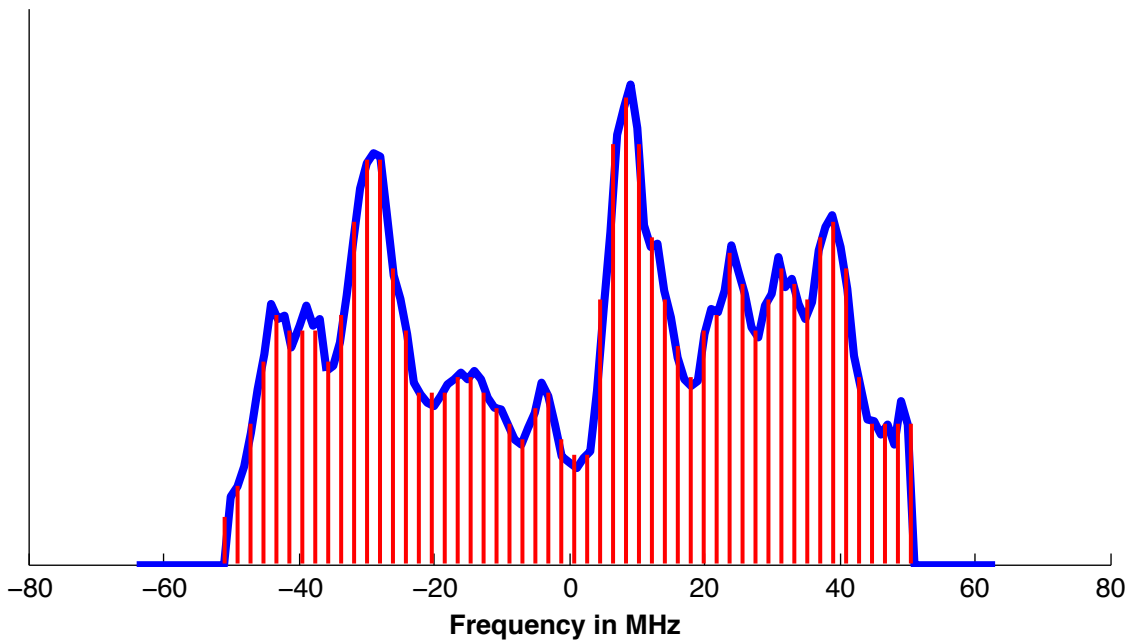

$$x(t) = \sum_i s_i[n] \mathrm{e}^{-j2\pi f_i t}$$

$$y(t) = \sum_i h_i s_i[n] \mathrm{e}^{-j2\pi f_i t}$$

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Multi-Carrier Modulation

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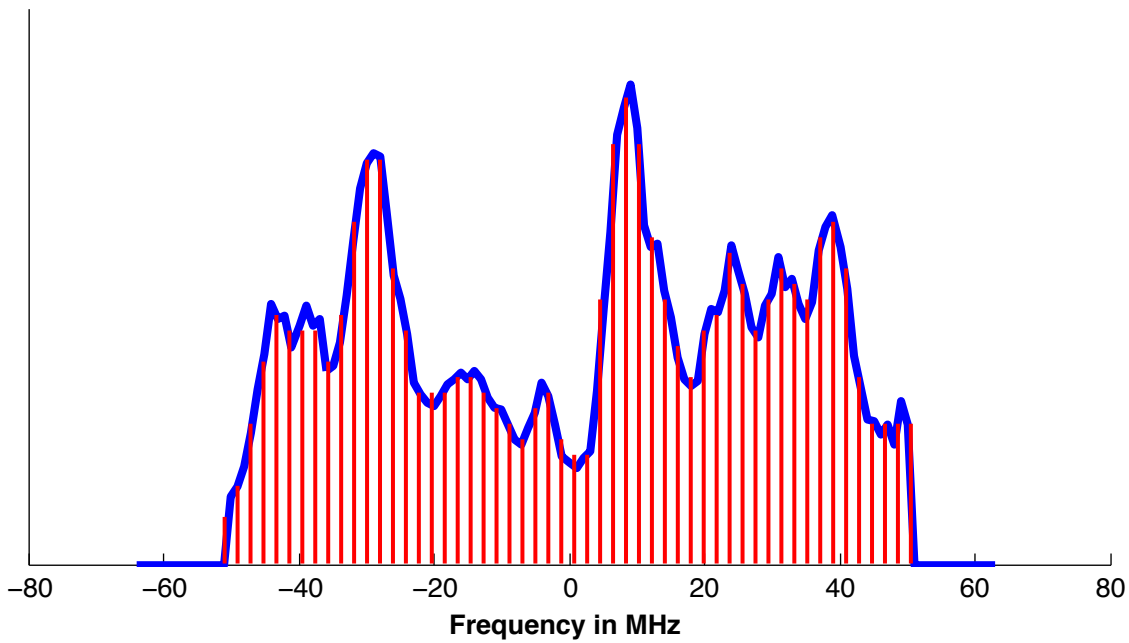

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Not That Simple!

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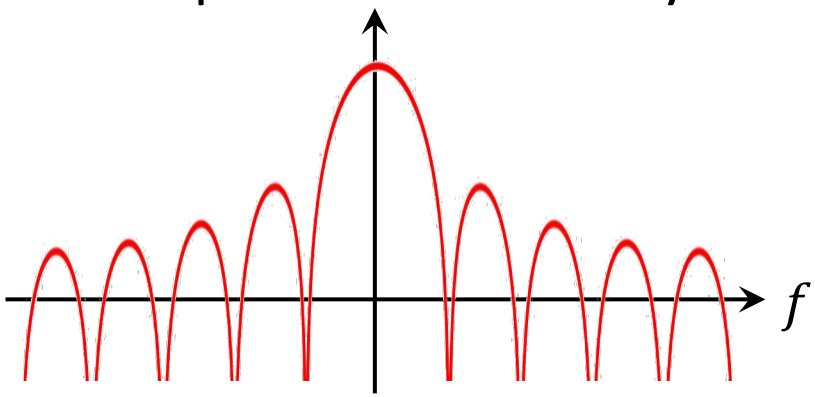
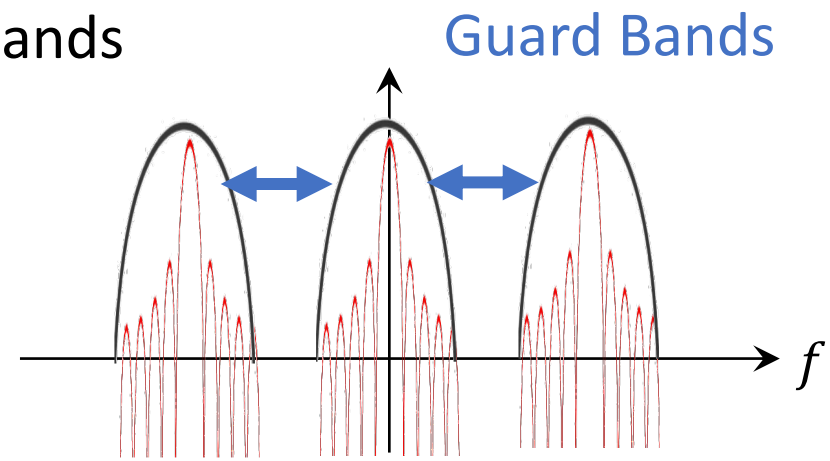




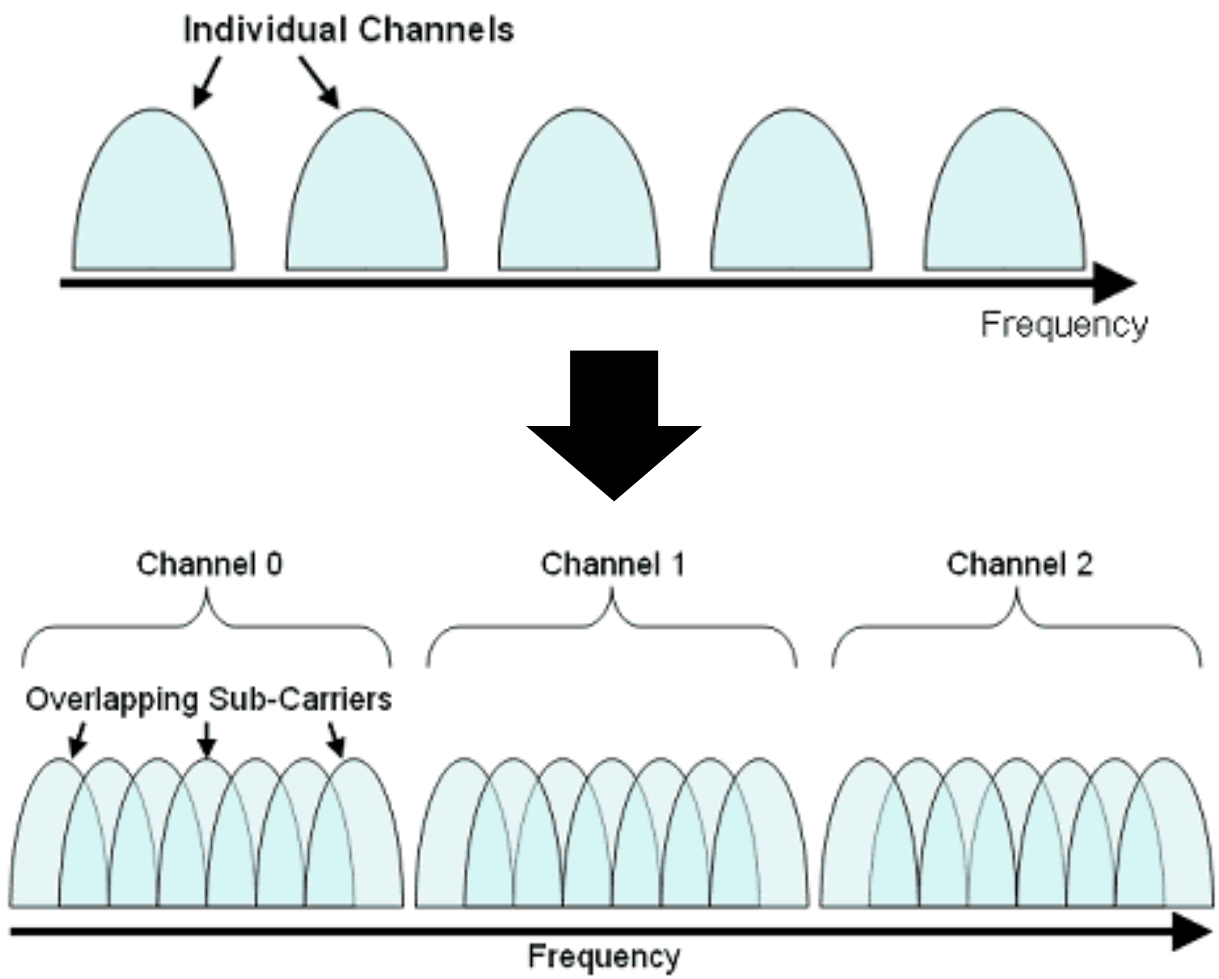
- Significant Leakage between adjacent subcarriers
- Need Guard Bands → Very inefficient!

Solution: Make the Sub-Carriers Orthogonal

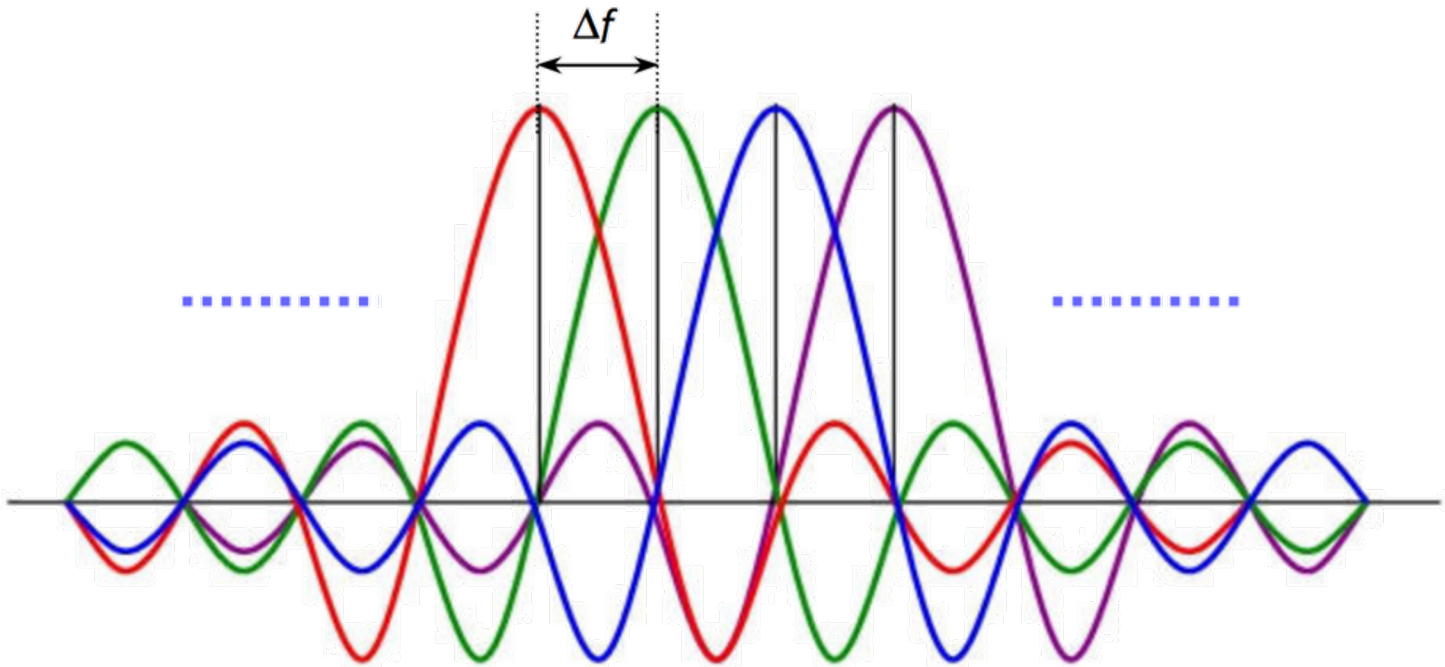
Multi-Carrier Modulation

Symbols modulated on multiple Sub-carrier frequencies

Make the Sub-Carriers Orthogonal



OFDM: Orthogonal Frequency Division Multiplexing

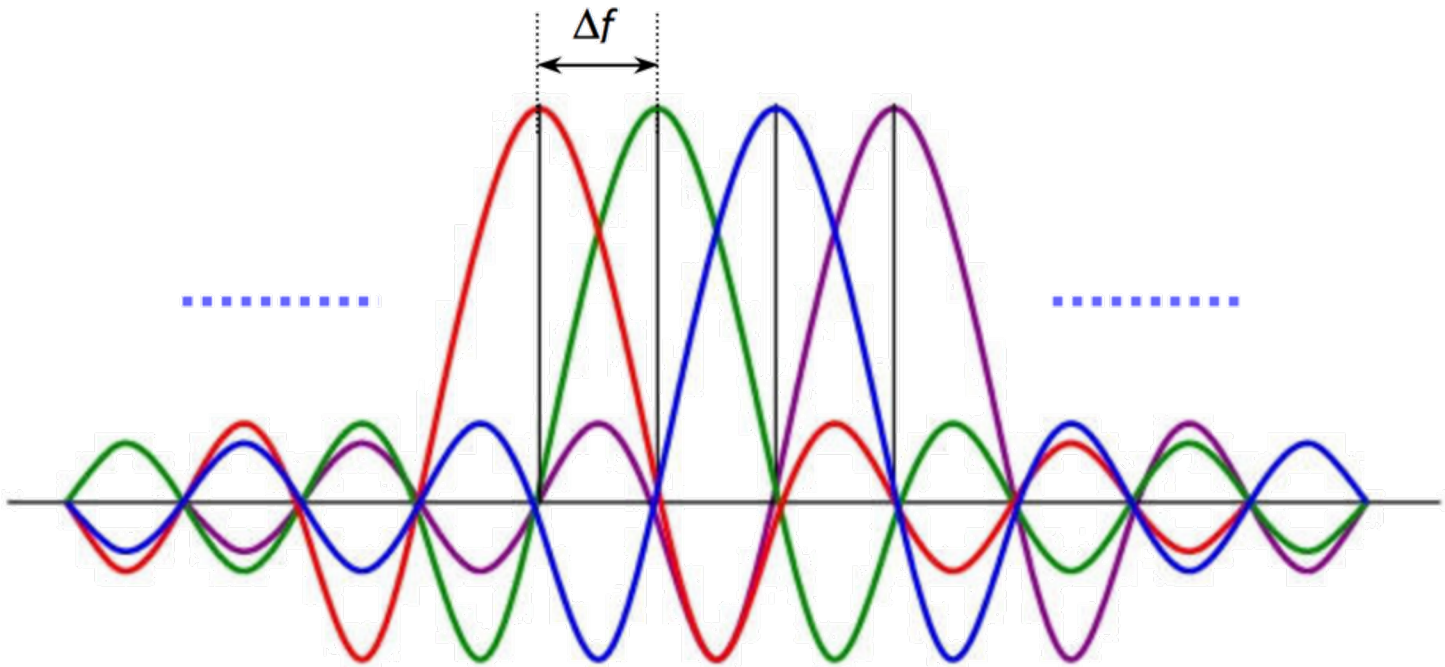


- Subcarriers are orthogonal: At the sub-carrier frequency, the sampled value has zero leakage from other subcarriers.

- Subcarrier separation can be very small, for N subcarriers and bandwidth B:

$$\Delta f = \frac{B}{N}$$

OFDM: Orthogonal Frequency Division Multiplexing



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How to Achieve This?

OFDM: Orthogonal Frequency Division Multiplexing

Use DFT: Discrete Fourier Transform

N-Point DFT: $X(f_i) = \frac{1}{N}\sum_{t=0}^{N-1} x(t)e^{-j\frac{2\pi f_i t}{N}}$

N-Point IDFT: $x(t) = \sum_{f_i=0}^{N-1} X(f_i)e^{j\frac{2\pi f_i t}{N}}$

Send symbols in Frequency Domain

$X(f_i) = s[n] \rightarrow$ Compute and transmit $x(t)$ using IDFT

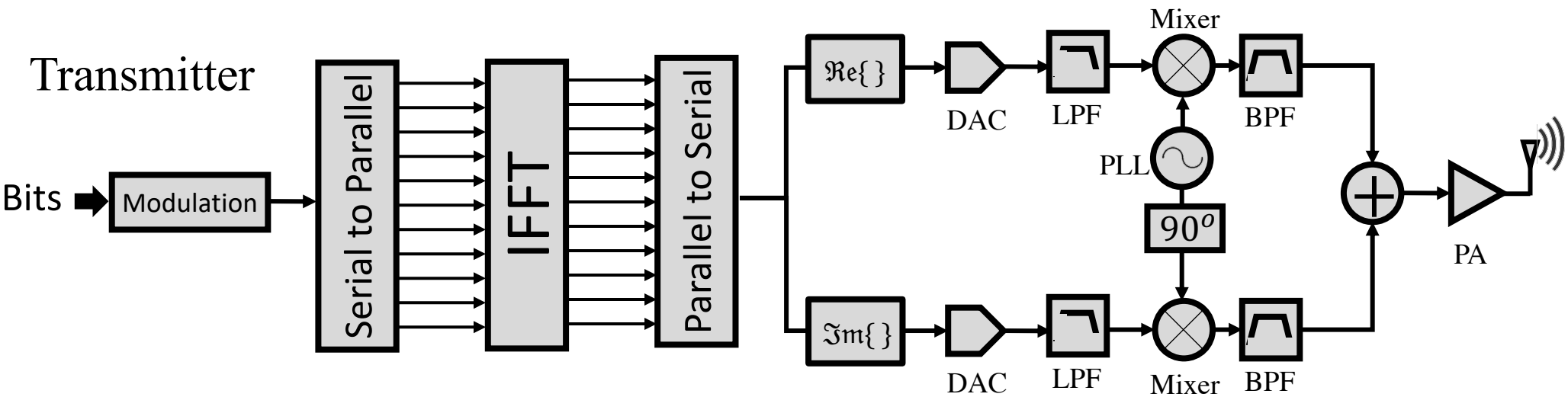
OFDM: Orthogonal Frequency Division Multiplexing

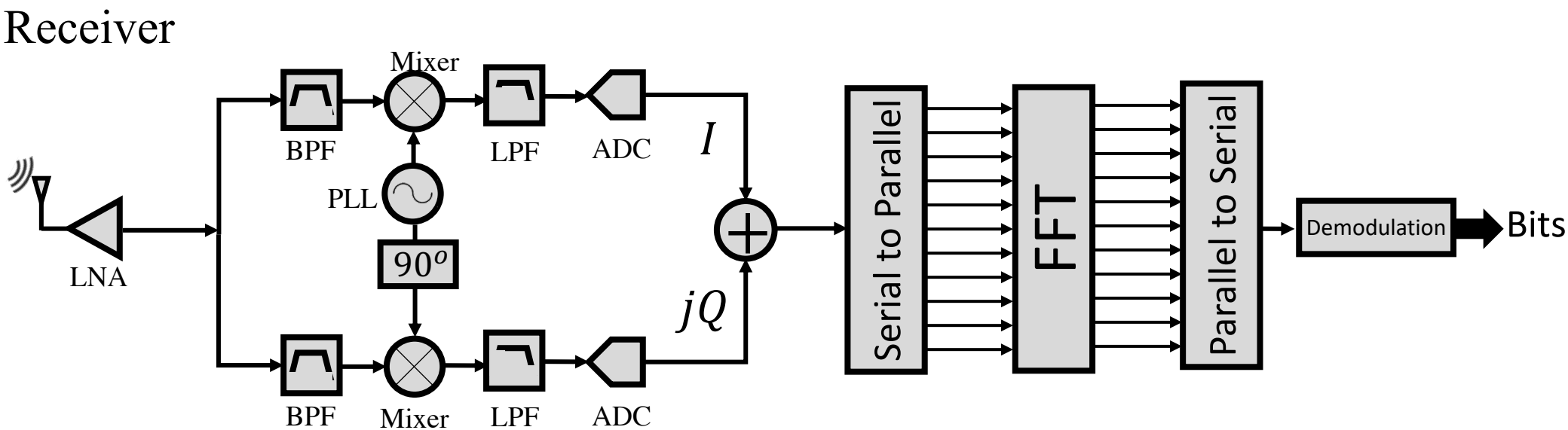
Send symbols in Frequency Domain

$X(f_i) = s[n] \rightarrow$ Compute and transmit $x(t)$ using IDFT

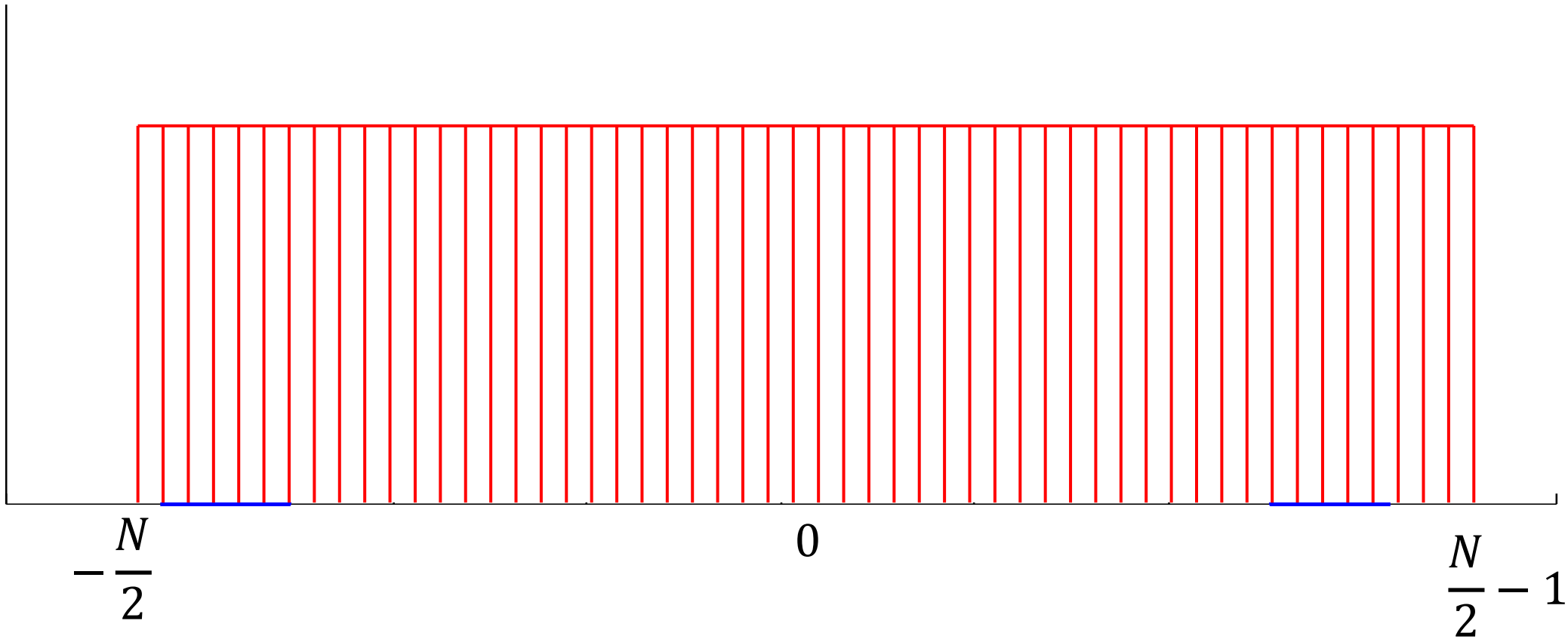
- Nsubcarrier → IDFT of length N
- Symbols $s[n]$ can come from any modulation: BPSK, QPSK, QAM…
- $x(t)$ is complex → need I & Q → No point using PAM or ASK …
- OFDM Symbol: N samples of $x(t)$ generated from the same modulated symbols using IDFT.
- OFDM Symbol Time: $T = N/B$ where B is the bandwidth.
- OFDM Frequency Bin Width: $\Delta f = 1/T = B/N$

OFDM: Orthogonal Frequency Division Multiplexing





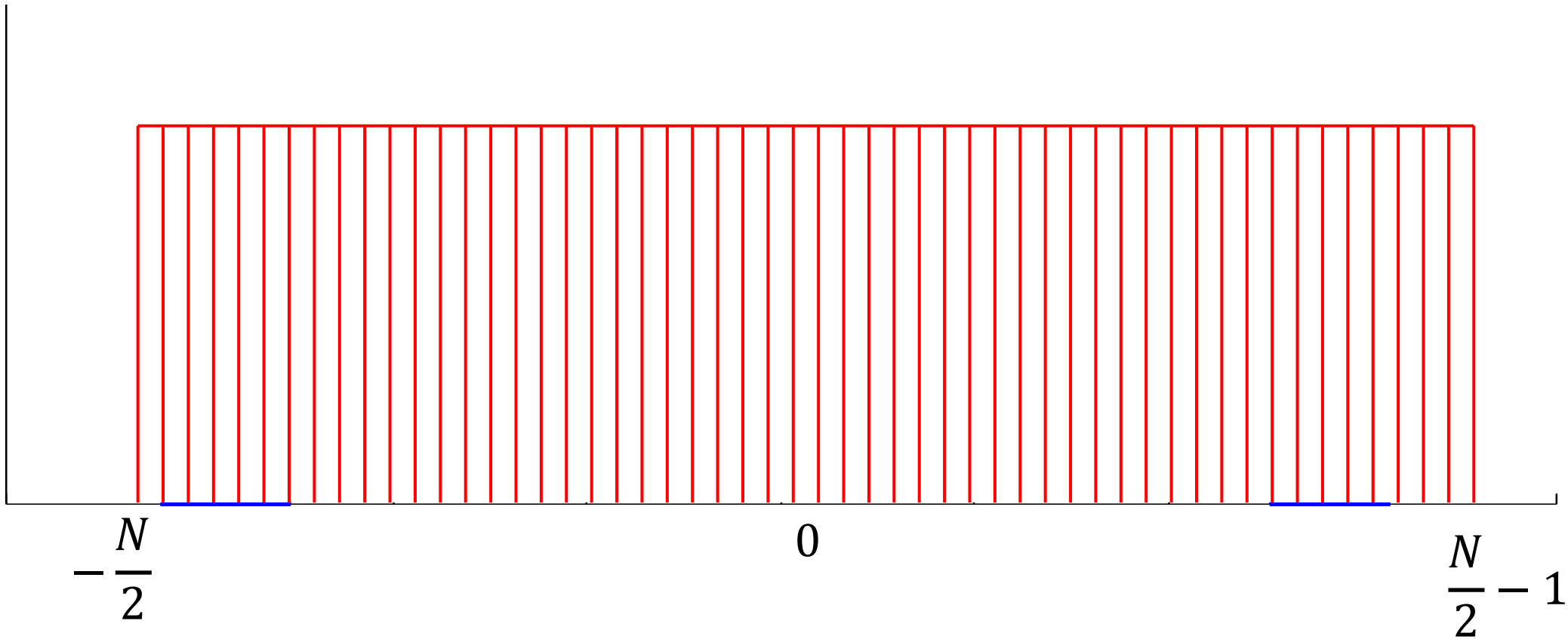
OFDM Symbol in Frequency Domain



- FFT can be represented 0 to $N-1$ or $N/2$ to $N/2\ -1$.
- OFDM Symbol created in digital baseband → 0 bin corresponds to DC

$$X(0) = \frac{1}{N}\sum_{t=0}^{N-1} x(t)e^{-j\frac{2\pi 0 t}{N}} = \frac{1}{N}\sum_{t=0}^{N-1} x(t) = DC$$

OFDM Symbol in Frequency Domain



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- DC of the circuits corrupts bits sent on the 0 bin → Do not use 0 bin

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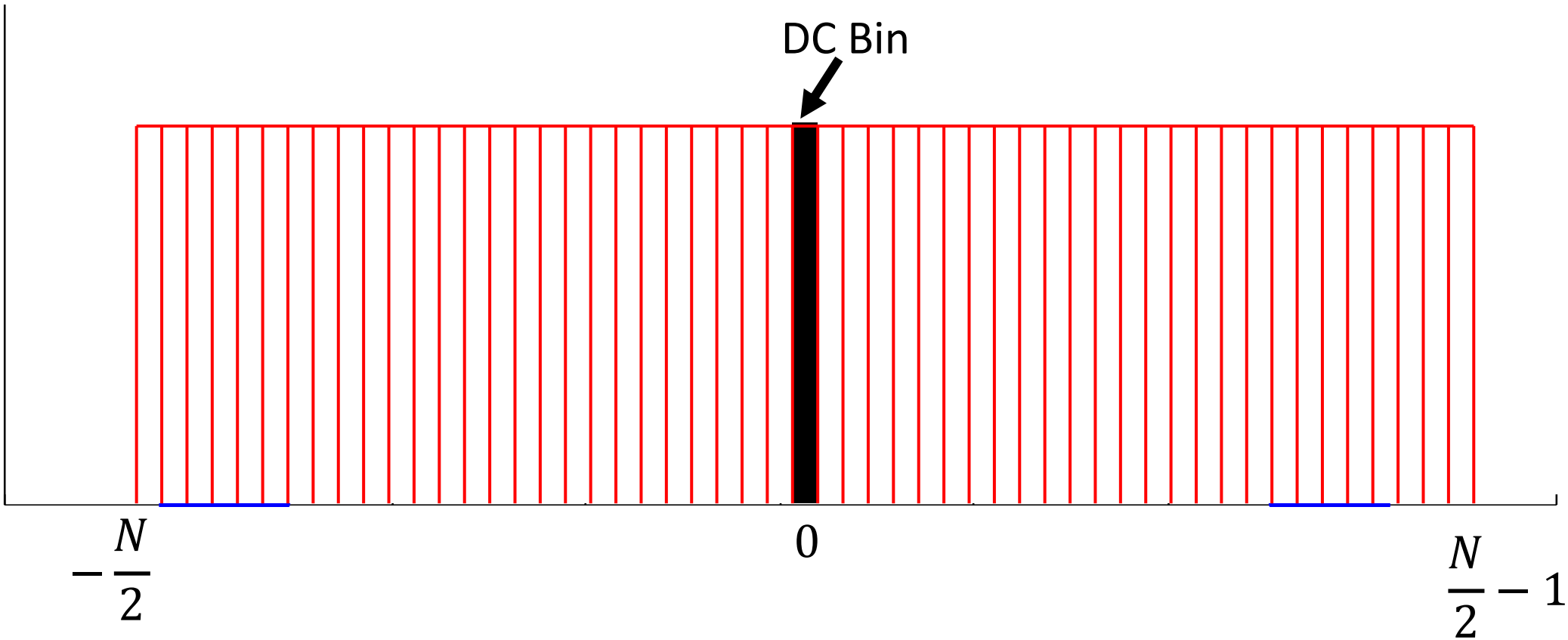

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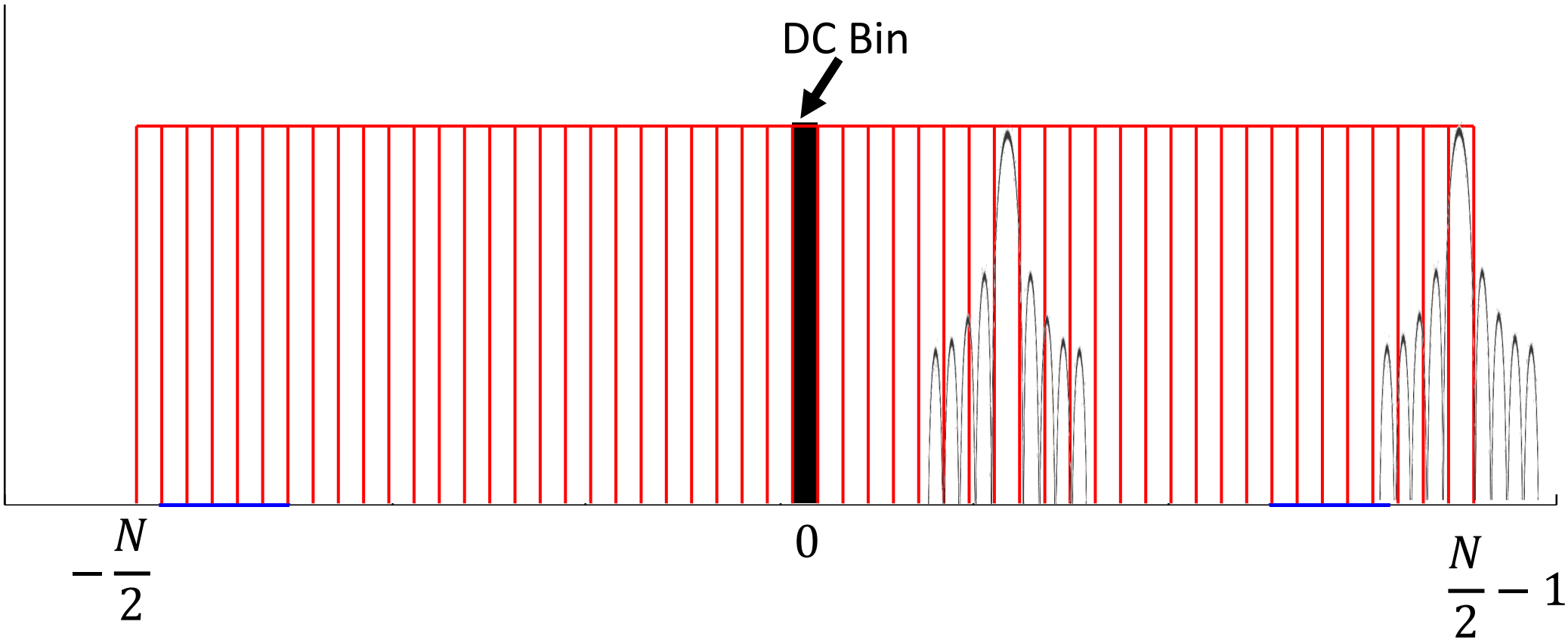
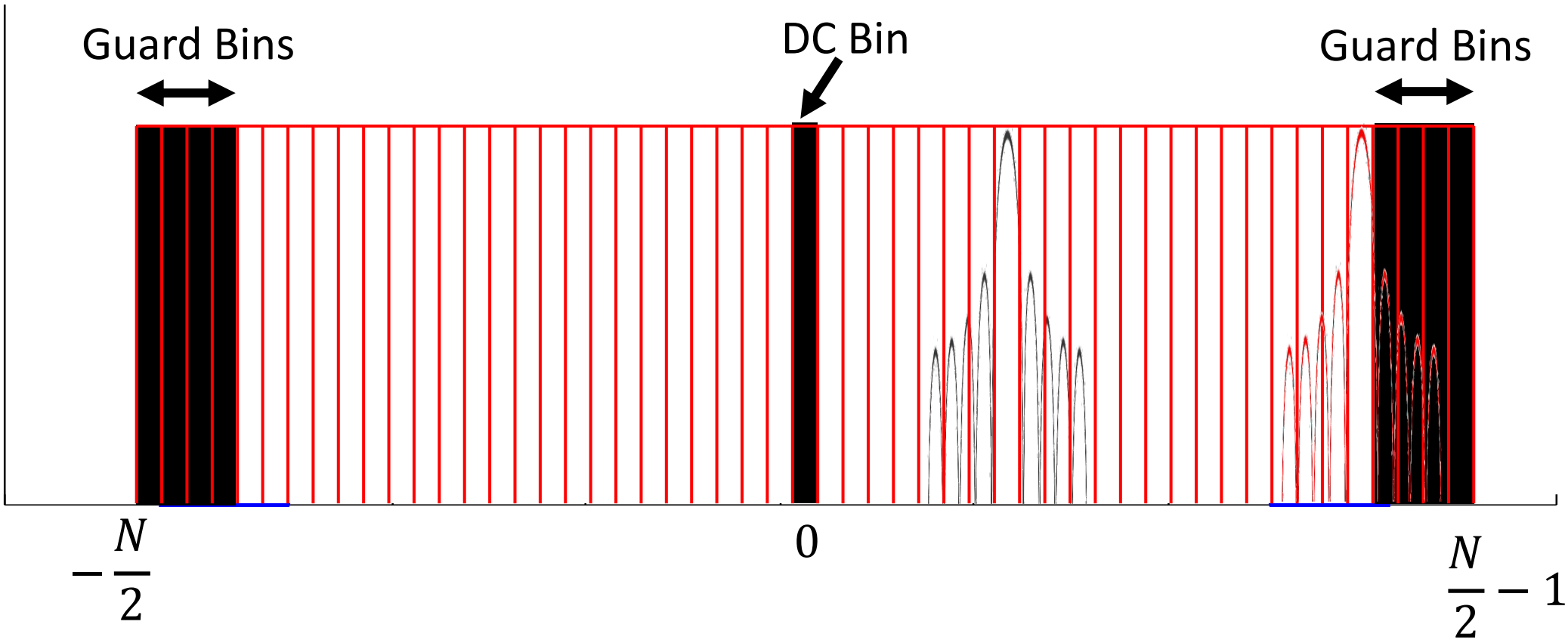


- Subcarriers orthogonal to each other but not to near by channels.
- Need Guard Bins at sides of the channel → Transmit nothing there

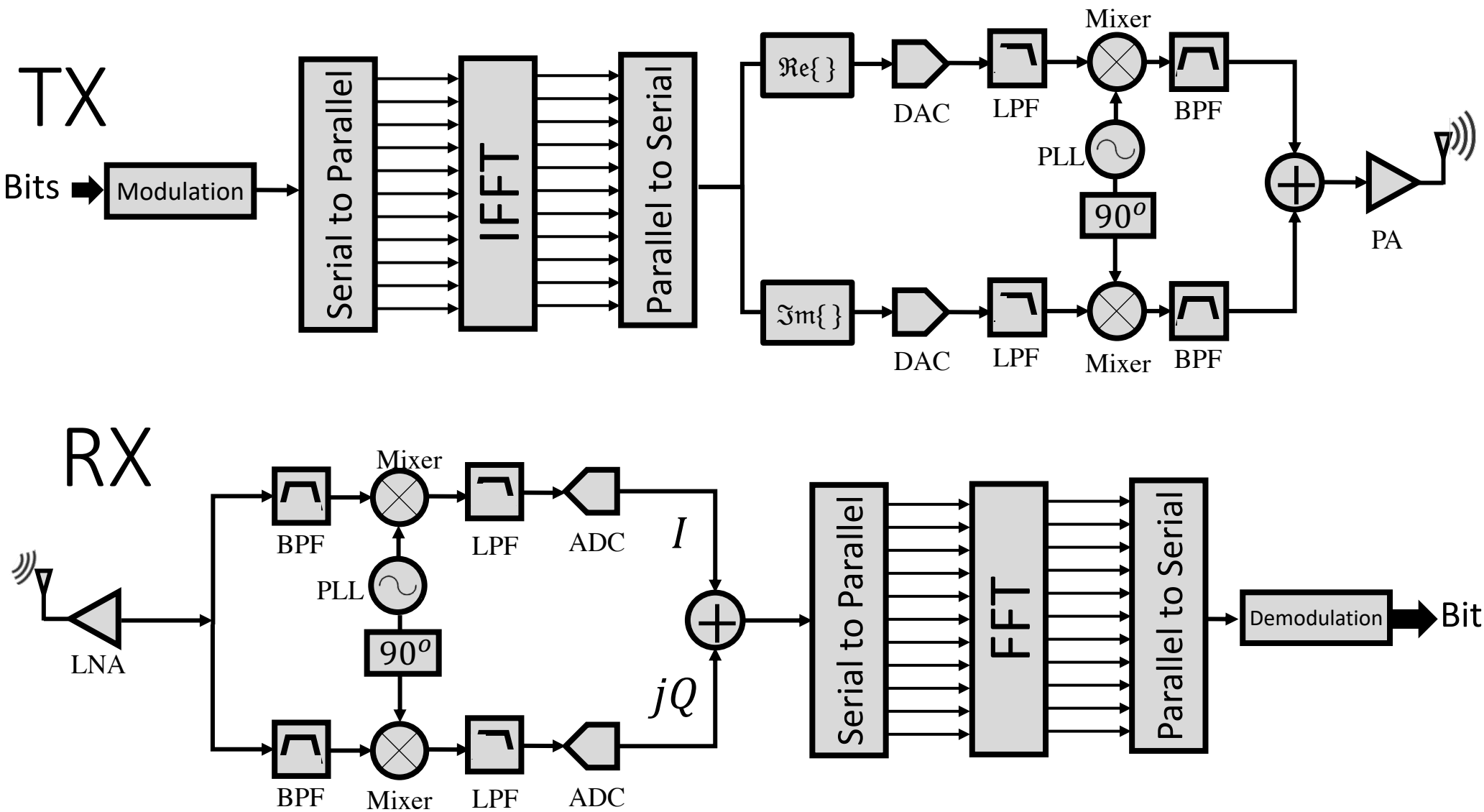
OFDM Symbol in Frequency Domain



- Subcarriers orthogonal to each other but not to near by channels.
- Need Guard Bins at sides of the channel → Transmit nothing there
- Reduce Number of Guard band from N to 2 → Very Spectrally Efficient

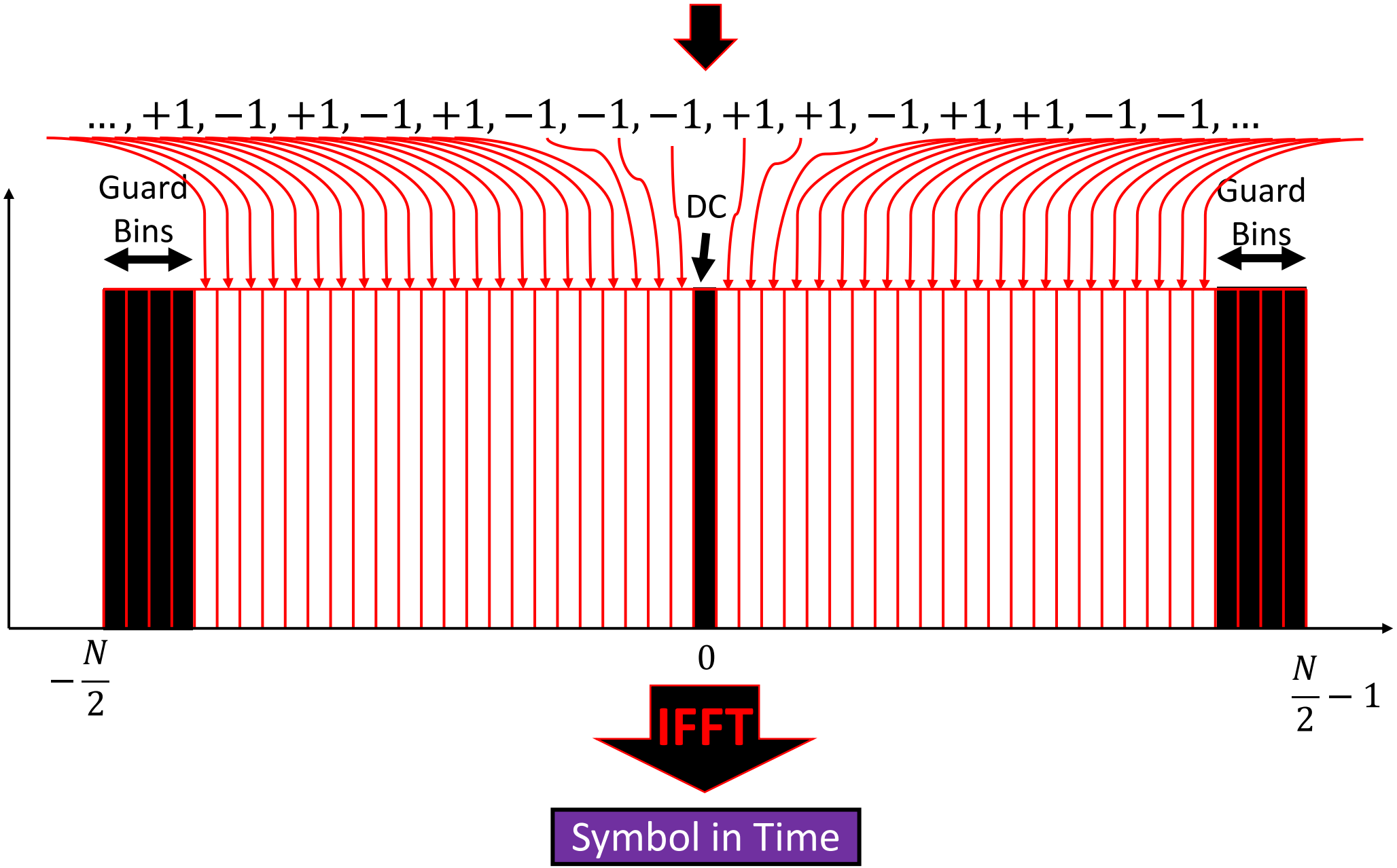
OFDM: Orthogonal Frequency Division Multiplexing

Transmit Symbols in Frequency Domain On Orthogonal Subcarriers



OFDM Symbol

Bits: 1 0 1 0 1 0 0 0 1 1 0 1 1 0 0



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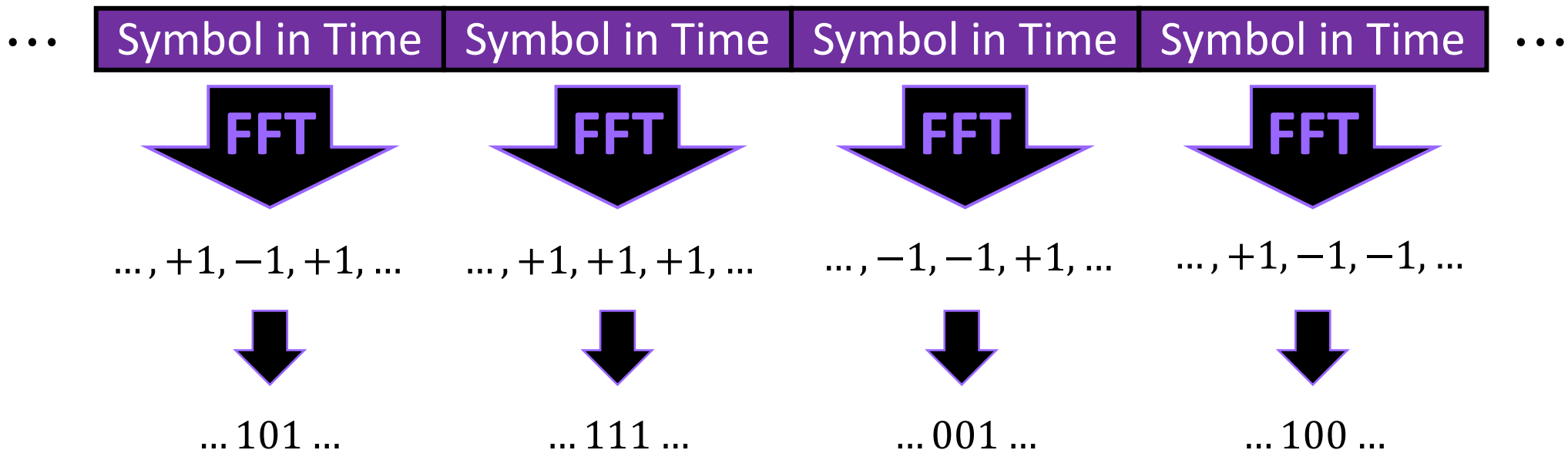
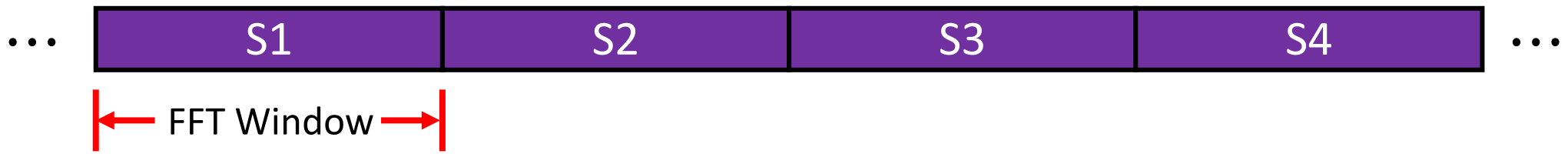


Not That Simple

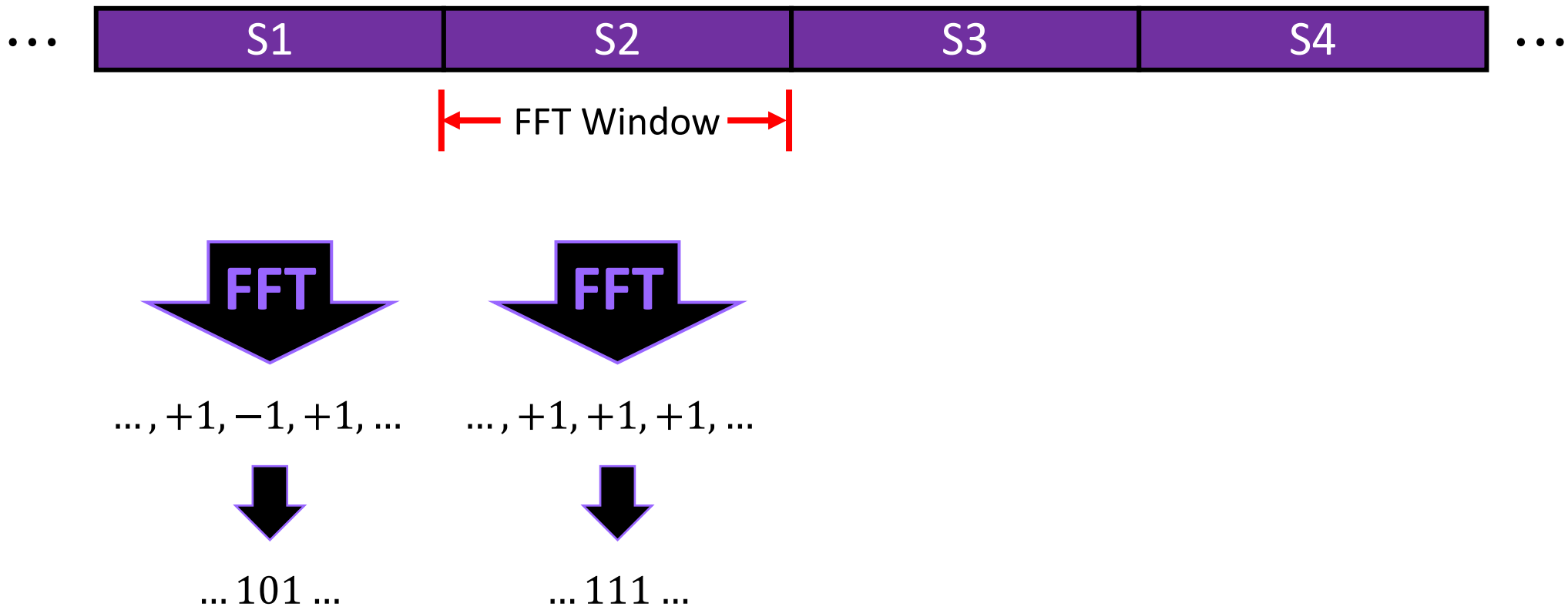
OFDM Symbol



$\ldots, +1, -1, +1, \ldots$

$\ldots 101 \ldots$

OFDM Symbol



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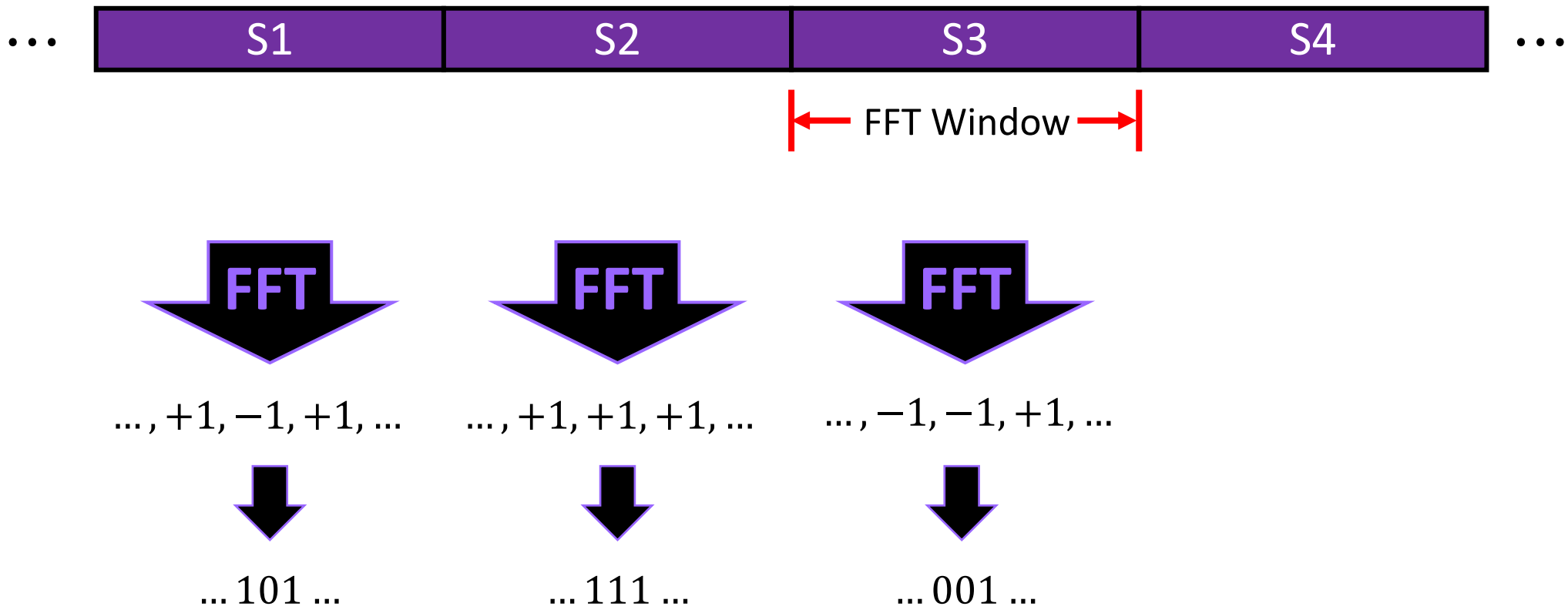

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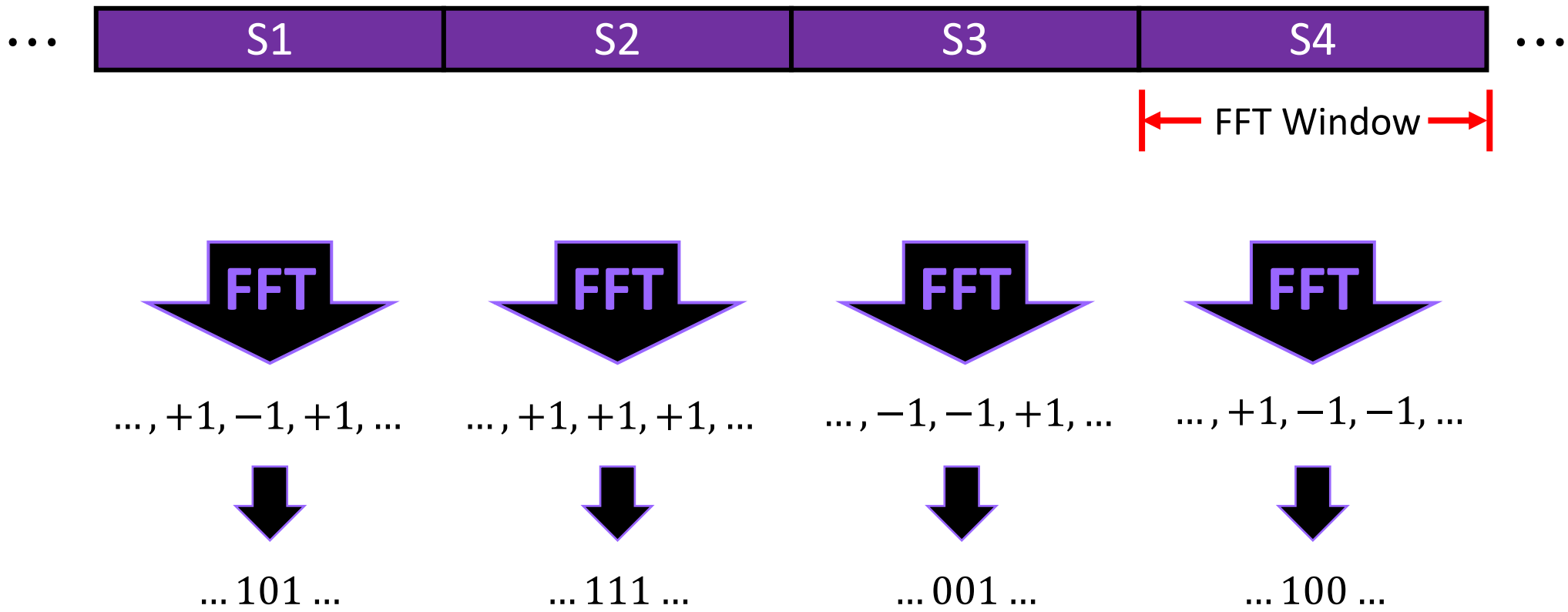
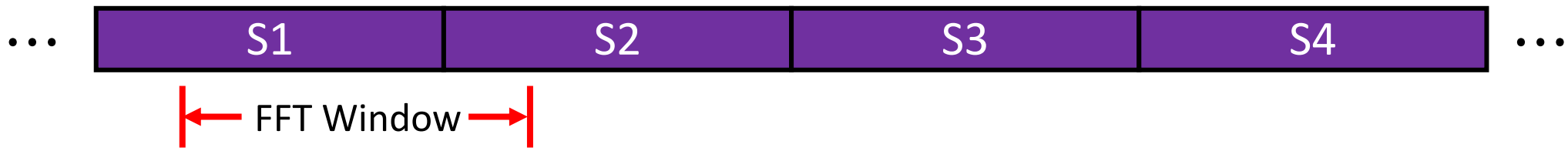


Assumes FFT window is perfectly aligned with symbol boundaries

OFDM Symbol



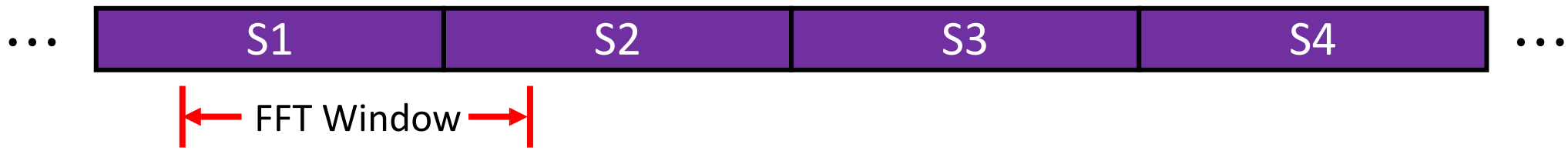
$\ldots, +0.5 + 1\mathrm{i}, -0.7 + 0.3\mathrm{i}, \ldots$

✕ Cannot decode!

FFT window is misaligned with symbol

Subcarriers are no longer orthogonal.

OFDM Cyclic Prefix



- DFT (FFT) assumes time samples are periodic of period N

- Circular Shift before taking FFT:

$$x[t] \rightarrow X[f]$$

$$x[t - \tau \bmod N] \rightarrow X[f]e^{-j\frac{2\pi f \tau}{N}}$$

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OFDM Cyclic Prefix

- Even if FFT window is misaligned, CP ensures that all samples come from the same symbol → Orthogonality is preserved!

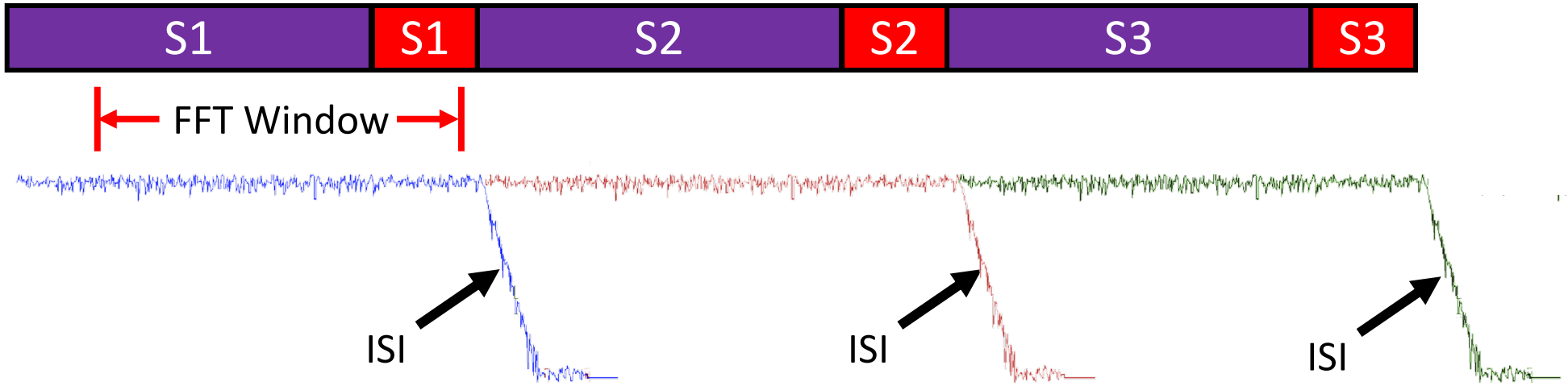
- Cyclic Prefix can be created by:
  - Take first few samples and append them to end of symbol.
  - Take last few samples and prefix them to beginning of symbol.

- Simple Phase Shift → Can be corrected by lumping with channel $H[f]$

OFDM Cyclic Prefix

Cyclic Prefix:

- Preserves orthogonality by allowing some misalignment in FFT Window
- Deals with Inter-Symbol-Interference



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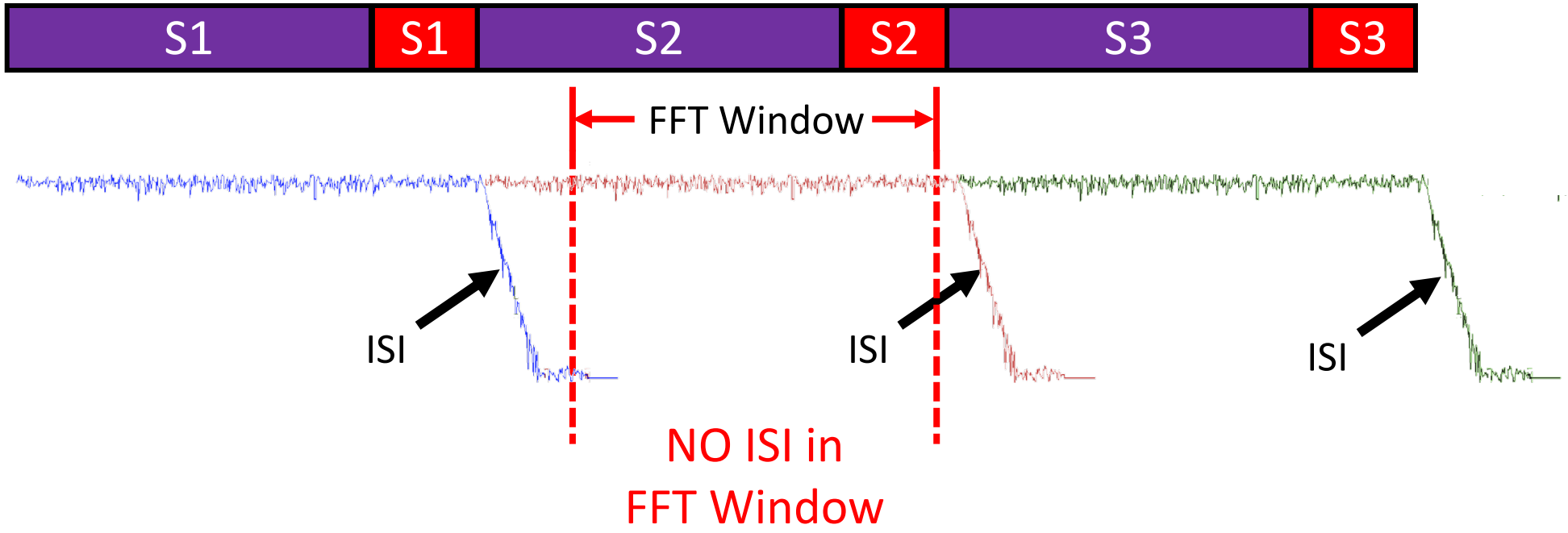

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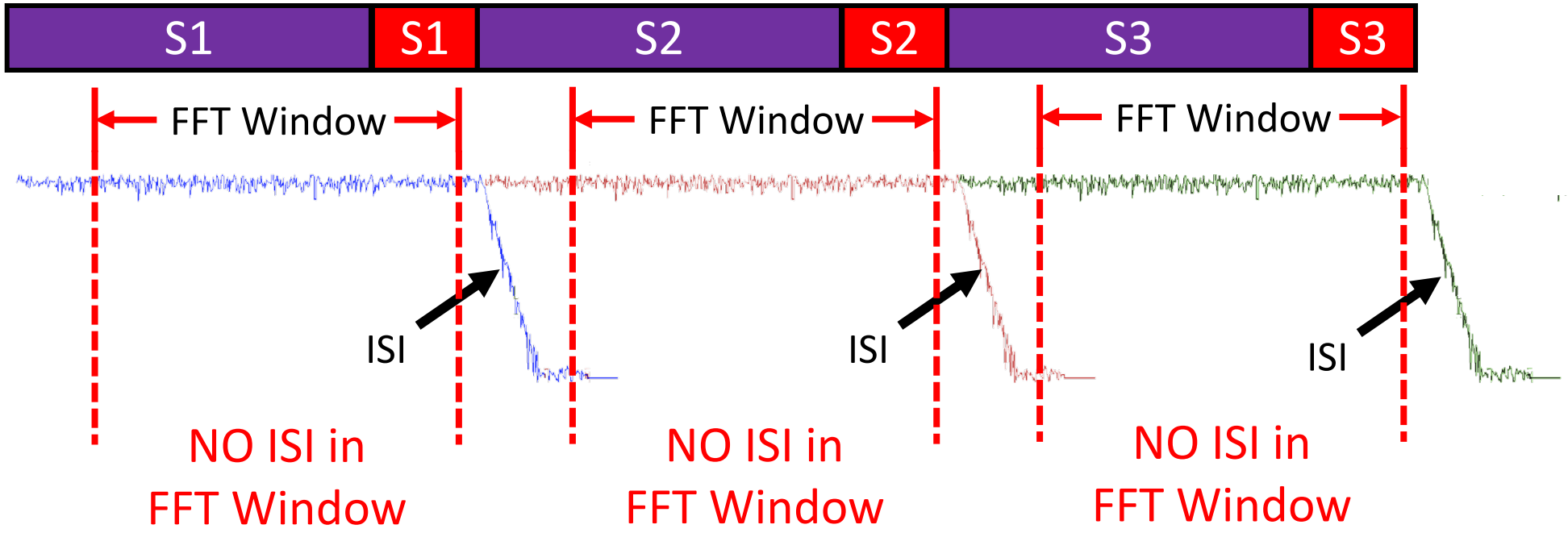


OFDM Cyclic Prefix

Cyclic Prefix:

+ Preserves orthogonality by allowing some misalignment in FFT Window

+ Deals with Inter-Symbol-Interference

− Overhead: Send $CP + N$ samples for every N samples

$$\text{Overhead} = \frac{CP}{CP + N}$$

$$\text{e.g. WiFi 802.11n: } N = 64, \text{CP} = 16 \rightarrow \text{Overhead} = 20\%$$

$$\text{e.g. LTE: } N = 1024, \text{CP} = 72 \rightarrow \text{Overhead} = 6.5\%$$

Progress so far?

A. I understand and I can follow

B. I understand most stuff but not everything

C. I can follow but I do not understand everything

D. I cannot follow and understood nothing about OFDM

OFDM Cyclic Prefix

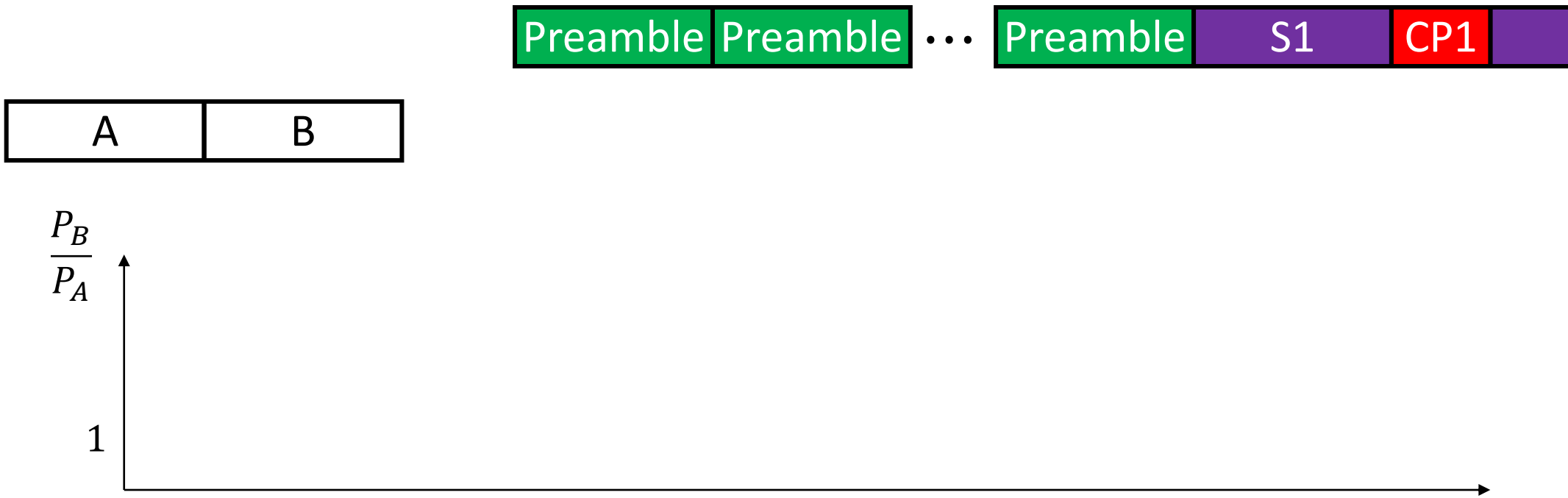
- Cyclic prefix is a not a bullet proof solution.
- Can still end up misaligned!
- Need a way to ensure we detect the beginning of the packet correctly.
- If we do, CP will ensure that even if we are not accurate, we can still decode.

OFDM Packet Detection

- Detect Beginning of packet to make sure we are within the CP
- Send Training Sequence: Preamble Symbols
- Preamble Symbols: Known Symbol Repeated at the beginning of packet

- No need for CP with preamble symbols

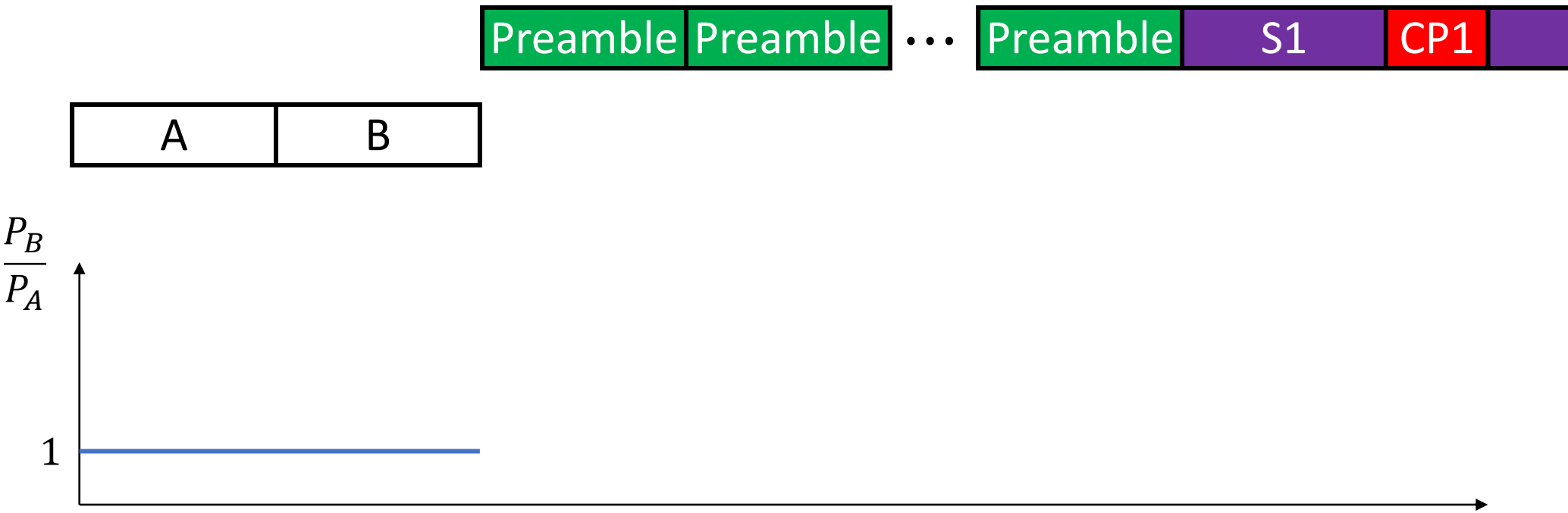
OFDM Packet Detection: Sliding Window



- Two windows of L $(2N)$ samples each.

- Compute: $\frac{P_B}{P_A} = \frac{\sum_{k=t+L}^{t+2L} |x[k]|^2}{\sum_{k=t}^{t+L} |x[k]|^2}$

OFDM Packet Detection: Sliding Window



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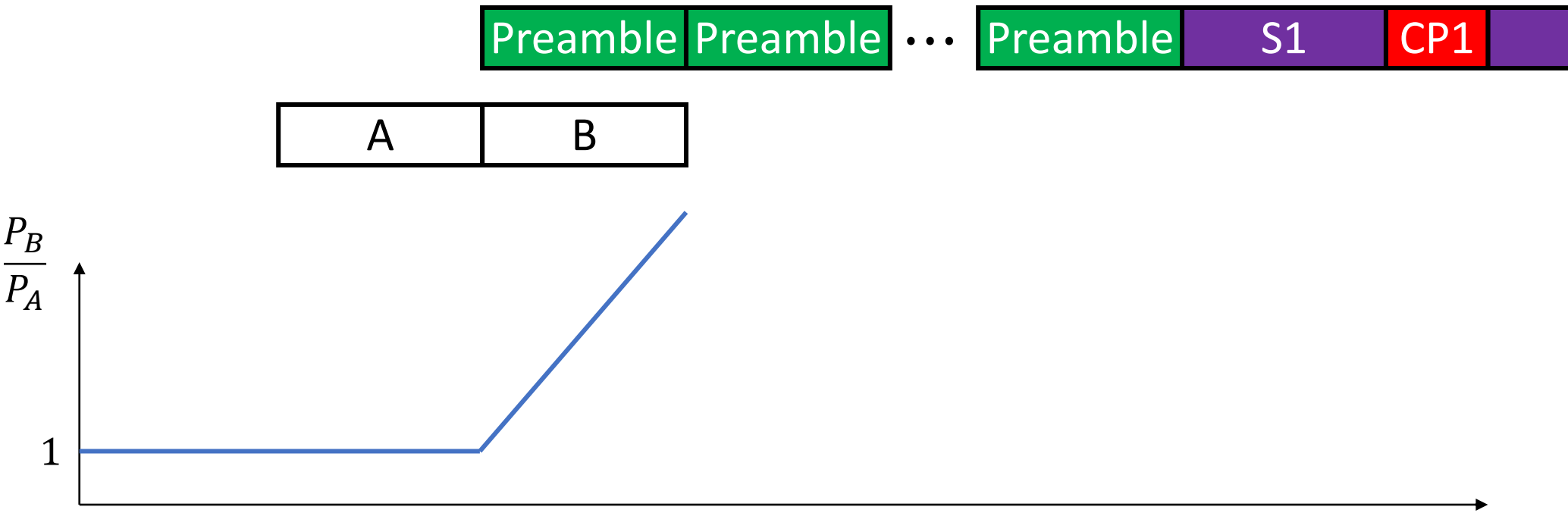

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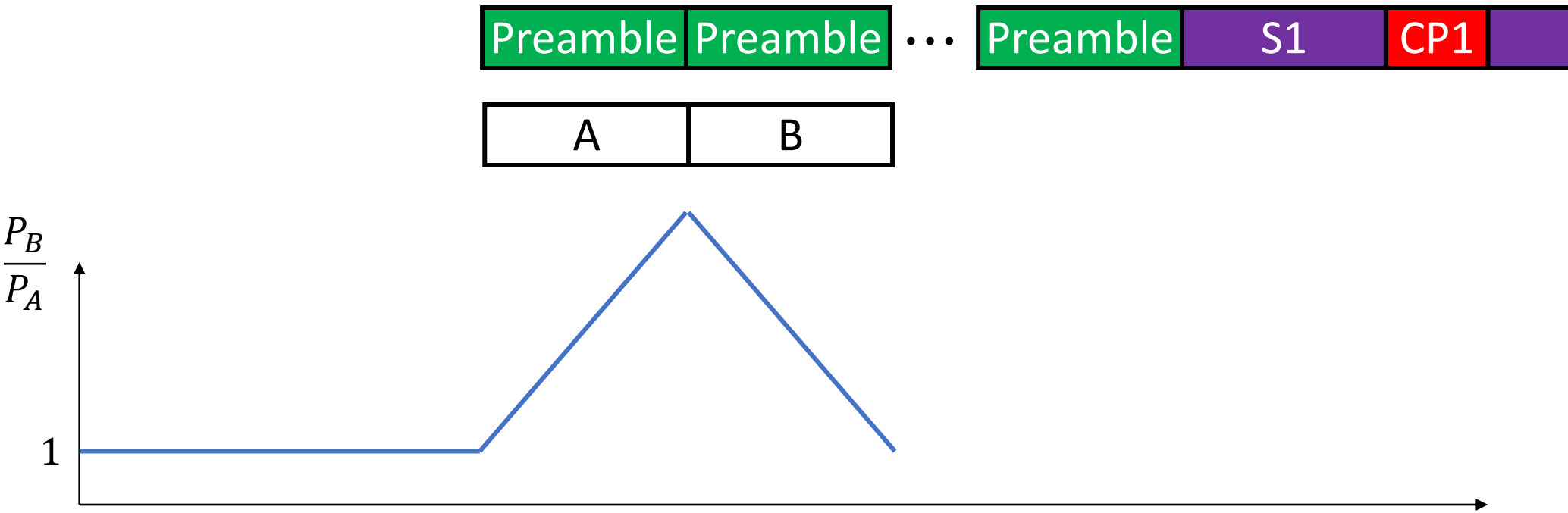

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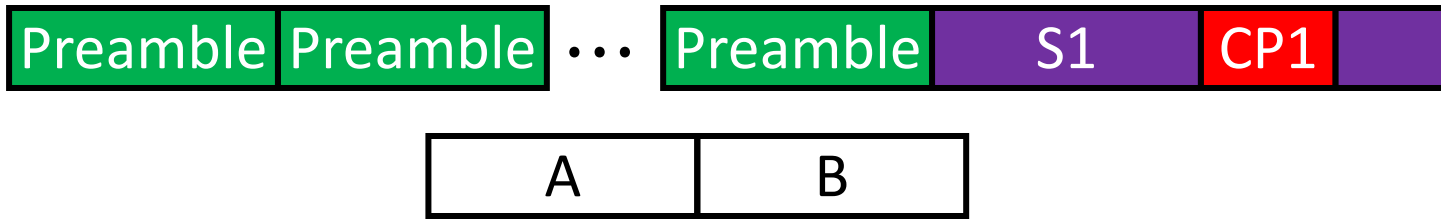

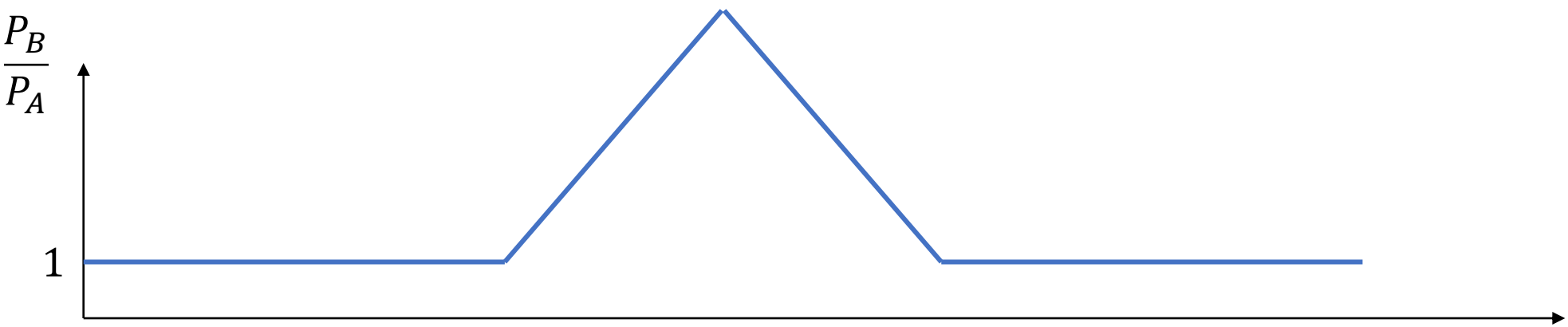

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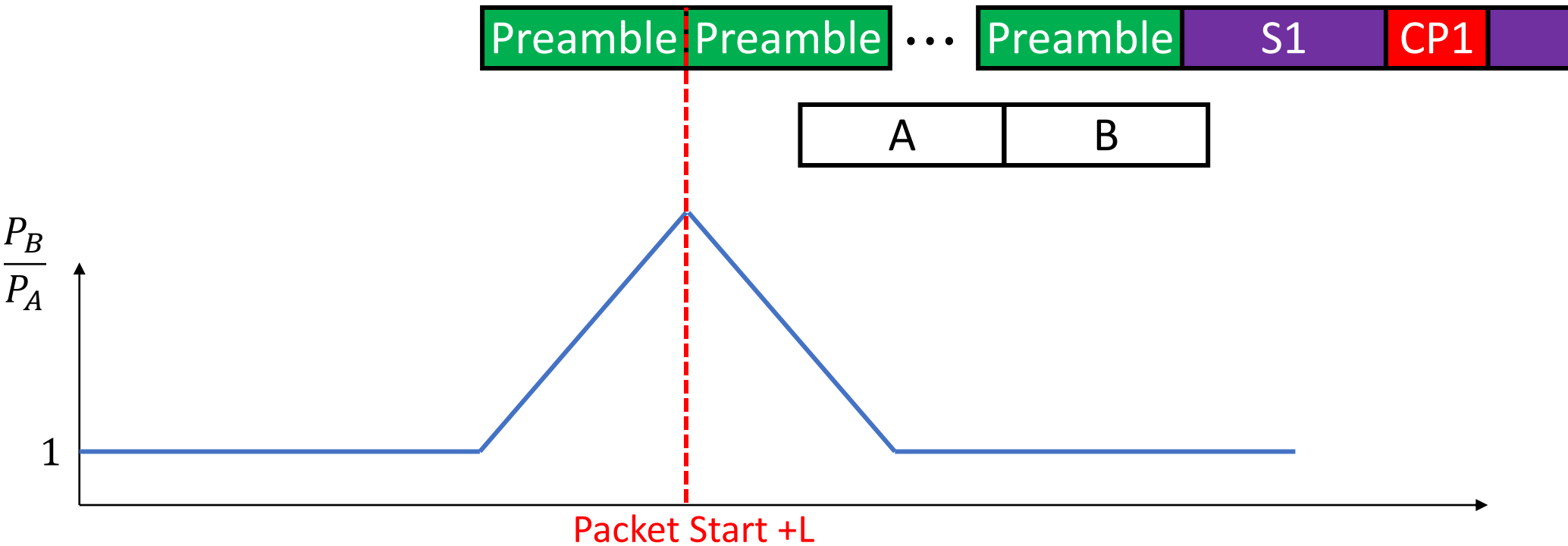
- Compute: $\frac{P_B}{P_A} = \frac{\sum_{k=t+L}^{t+2L} |x[k]|^2}{\sum_{k=t}^{t+L} |x[k]|^2}$

OFDM Packet Detection: Sliding Window



- Two windows of L samples each.

- Compute: $\frac{P_B}{P_A} = \frac{\sum_{k=t+L}^{t+2L} |x[k]|^2}{\sum_{k=t}^{t+L} |x[k]|^2}$

Previous Lecture:

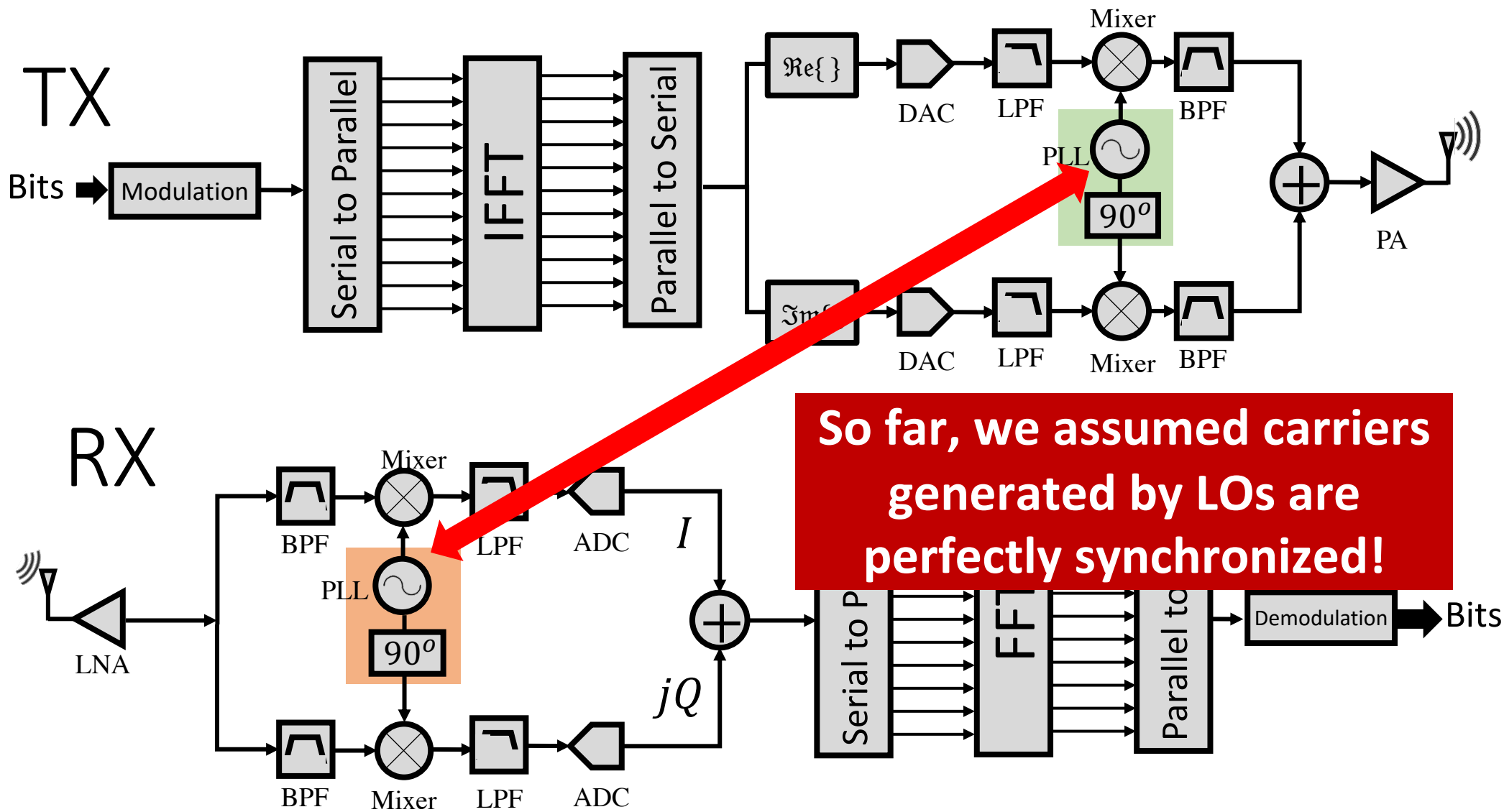
- ✓ Pulse Shaping
- ✓ Matched Filter
- ✓ Multipath Channel
- ✓ Channel Estimation & Correction
- ✓ Narrowband vs. Wideband Channels
- ✓ Channel Equalization

This Lecture:

- ✓ Multi-Carrier Modulation
- ✓ Orthogonal Frequency Division Multiplexing (OFDM)
- ✓ OFDM Time Synchronization
- ❑ OFDM Frequency Synchronization
- ❑ OFDM Channel Estimation & Correction
- ❑ OFDM Phase Tracking

OFDM: Orthogonal Frequency Division Multiplexing

Transmit Symbols in Frequency Domain On Orthogonal Subcarriers



So far, we assumed carriers generated by LOs are perfectly synchronized!

Carrier Frequency Offset

$x(t) \Rightarrow x(t) \times e^{-j2\pi f_c t} \Rightarrow h(t) * x(t)e^{-j2\pi f_c t} \Rightarrow h(t) * x(t)e^{-j2\pi f_c t} \times e^{j2\pi f_c t}$

$\Rightarrow h(t) * x(t)$

$\Rightarrow y(t) = h(t) * x(t) + v(t)$

Assumes TX & RX perfectly synched

Carrier Frequency Offset

$x(t) \Rightarrow x(t) \times e^{-j2\pi f_c t} \Rightarrow h(t) * x(t) e^{-j2\pi f_c t} \Rightarrow h(t) * x(t) e^{-j2\pi f_c t} \times e^{j2\pi f_c' t}$

TX & RX are not synched $\Rightarrow h(t) * x(t)\, e^{-j2\pi \Delta f_c t}$

CFO: $\Delta f_c = f_c - f_c'$ $\Rightarrow y(t) = h(t) * x(t) e^{-j2\pi \Delta f_c t} + v(t)$

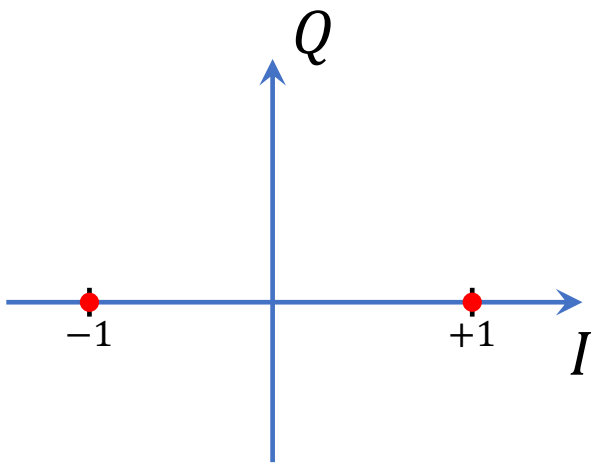
Phase changes with time!

Carrier Frequency Offset

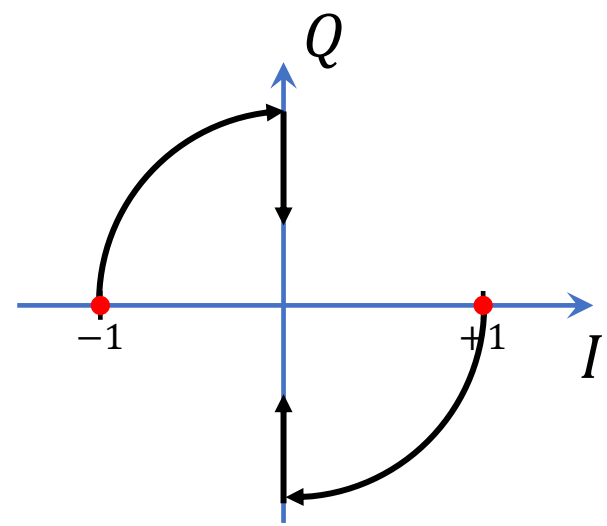
Consider BPSK Modulation.

$$0 \rightarrow -1$$

$$1 \rightarrow +1$$



$x(t)$



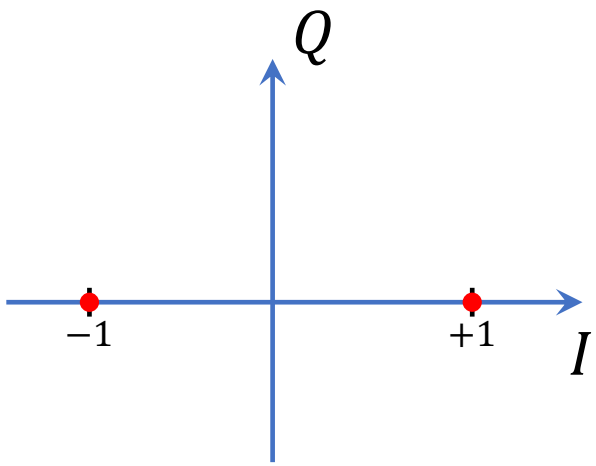
$h\, x(t-\tau) e^{-j2\pi\Delta f_c t} + v(t)$

Carrier Frequency Offset

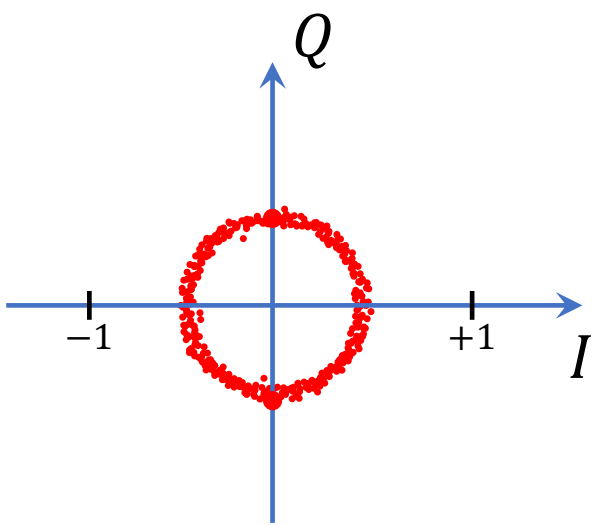
Consider BPSK Modulation.

$$0 \rightarrow -1$$

$$1 \rightarrow +1$$



$x(t)$

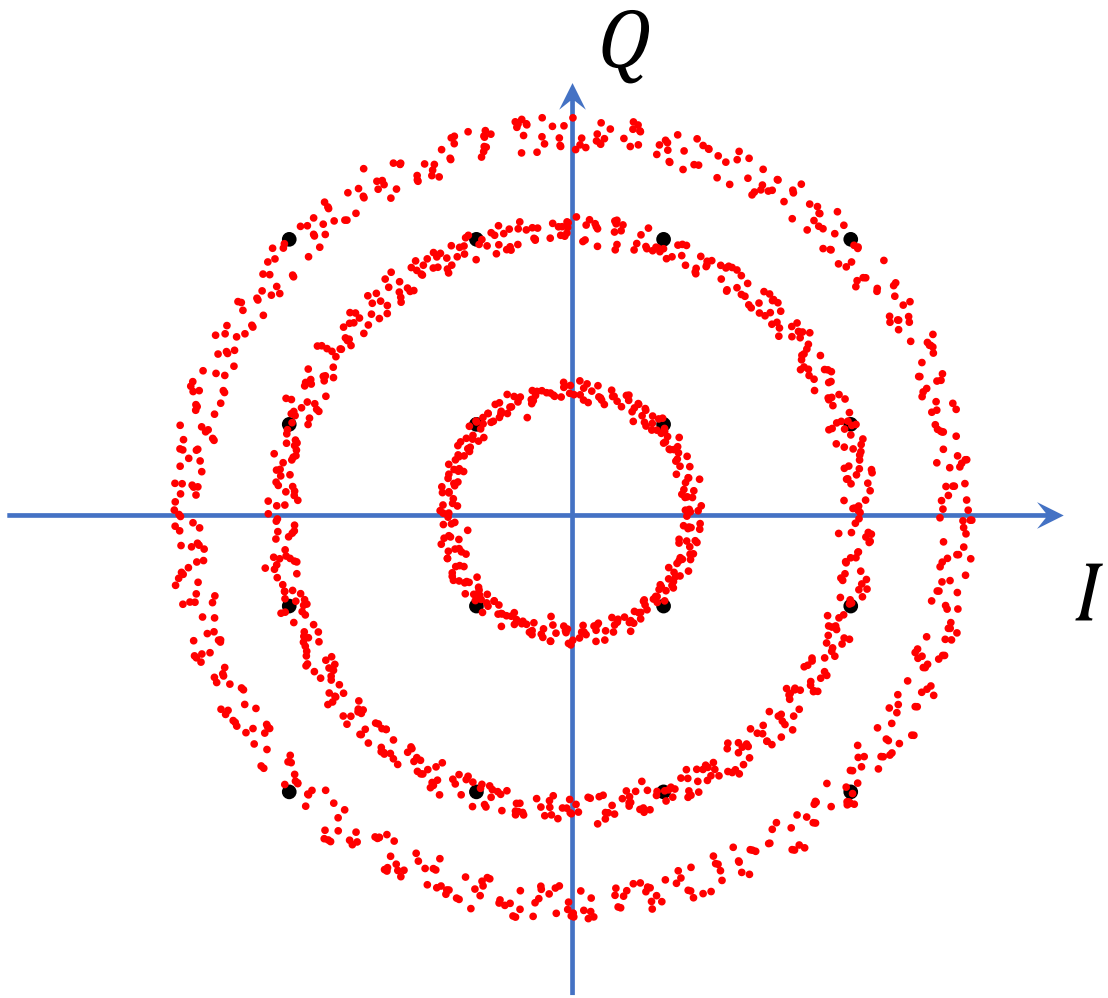


$h\, x(t-\tau) e^{-j2\pi\Delta f_c t} + v(t)$

Impossible to Decode!

Carrier Frequency Offset

Consider 16 QAM Modulation



Need to estimate and correct CFO to decode!

OFDM CFO Estimation & Correction

- Use Preamble to estimate CFO

$$y_1(t) = x(t)e^{-j2\pi\Delta f_c t}$$

$$y_2(t) = x(t)e^{-j2\pi\Delta f_c (t+NT_s)}$$

OFDM CFO Estimation & Correction

- Use Preamble to estimate CFO

$$y_1[n] = x[n]e^{-j2\pi\Delta f_c n T_s}$$

$$y_2[n] = x[n]e^{-j2\pi\Delta f_c (nT_s + NT_s)}$$

- Compute: $A = \sum_{t=1}^{N} y_1[n] y_2^*[n] = \sum_{t=1}^{N} x[n]x^*[n]e^{j2\pi\Delta f_c N T_s}$

$$= e^{j2\pi\Delta f_c N T_s} \sum_{t=1}^{N} |x[n]|^2 \quad \Rightarrow \quad \Delta f_c = \frac{\angle A}{2\pi N T_s}$$

OFDM CFO Estimation & Correction

Preamble	Preamble	⋯	Preamble	S1	CP1	S2	CP2	⋯

- Use Preamble to estimate CFO

$$y_1[n] = x[n]e^{-j2\pi\Delta f_c n T_s}$$

$$y_2[n] = x[n]e^{-j2\pi\Delta f_c (nT_s + NT_s)}$$

- Compute: $A = \sum_{t=1}^{N} y_1[n] y_2^*[n]$ ➡ $\Delta f_c = \frac{\angle A}{2\pi N T_s}$

- Correct CFO: $y[n] \times e^{j2\pi\Delta f_c n T_s}$

We use the following equation to estimate CFO: $\Delta f_c = \frac{\angle A}{2\pi N T_s}$.

Suppose f_c = 5 GHz, the bandwidth = 10 MHz and the clock precision is 20ppm. For what values of N will the above equation estimate the CFO incorrectly?

A. N < 10

B. N < 20

C. N > 50

D. N < 50

- Equation give wrong result when the phase of A wraps around 2π! We need: $|\angle A| \leq \pi$
- Δf_c = 5 GHz × 20/1000000 = 100 kHz
- $T_s = 1/10\text{MHz} = 0.1\mu s$

$$\Delta f_c \leq \frac{1}{2NT_s}$$

OFDM Channel Estimation

Preamble	Preamble	$\cdots$	Preamble	S1	CP1	S2	CP2	$\cdots$

- Use Preamble to estimate the channel

$$y(t) = h(t) * x(t) \leftrightarrow Y(f) = H(f)\,X(f)$$

- Send $X(f)$: $-1, +1, -1, -1, -1, +1, \ldots$
- Receive: $-H(1), H(2), -H(3), -H(4), -H(5), H(6), \ldots$
- Estimate: $\widetilde{H}(f) = \dfrac{Y(f)}{X(f)}$
- Use two preambles to average noise: $\widetilde{H}(f) = \dfrac{Y_1(f) + Y_2(f)}{2\,X(f)}$

Phase Tracking

So Far: Estimated and Corrected For Coarse Value of CFO

- **Residual CFO:**

$$y(t) = h(t)\, x(t) e^{-j2\pi\Delta f_c t} + v(t)$$

$$\Delta f_c = df_c + \delta f_c$$

Coarse CFO Residual CFO

We estimated and corrected for coarse CFO!

Even small residual can accumulate over time to create large phase: $e^{-j2\pi\delta f_c t}$

Need to track the phase

Phase Tracking

- Residual CFO (Carrier Frequency Offset)
- Residual SFO (Sampling Frequency Offset)

Phase Tracking

- Residual CFO (Carrier Frequency Offset)

- Residual SFO (Sampling Frequency Offset)

When we sample the signal there is a residual sampling offset: $n\delta T_s$

$$y[n] = x[n + n\delta T_s]e^{-j2\pi\delta f_c n T_s}$$

$$= \sum_{f=0}^{N-1} X[f]e^{j\frac{2\pi f(n+n\delta T_s)}{N}} e^{-j2\pi\delta f_c n T_s}$$

$$Y[f] = X[f]e^{j\frac{2\pi f n\delta T_s}{N} - 2\pi\delta f_c n T_s}$$

Phase Tracking

- Residual CFO (Carrier Frequency Offset)
- Residual SFO (Sampling Frequency Offset)

$$Y_1[f] = X_1[f]e^{j\frac{2\pi f n \delta T_s}{N} - 2\pi\delta f_c n T_s}$$

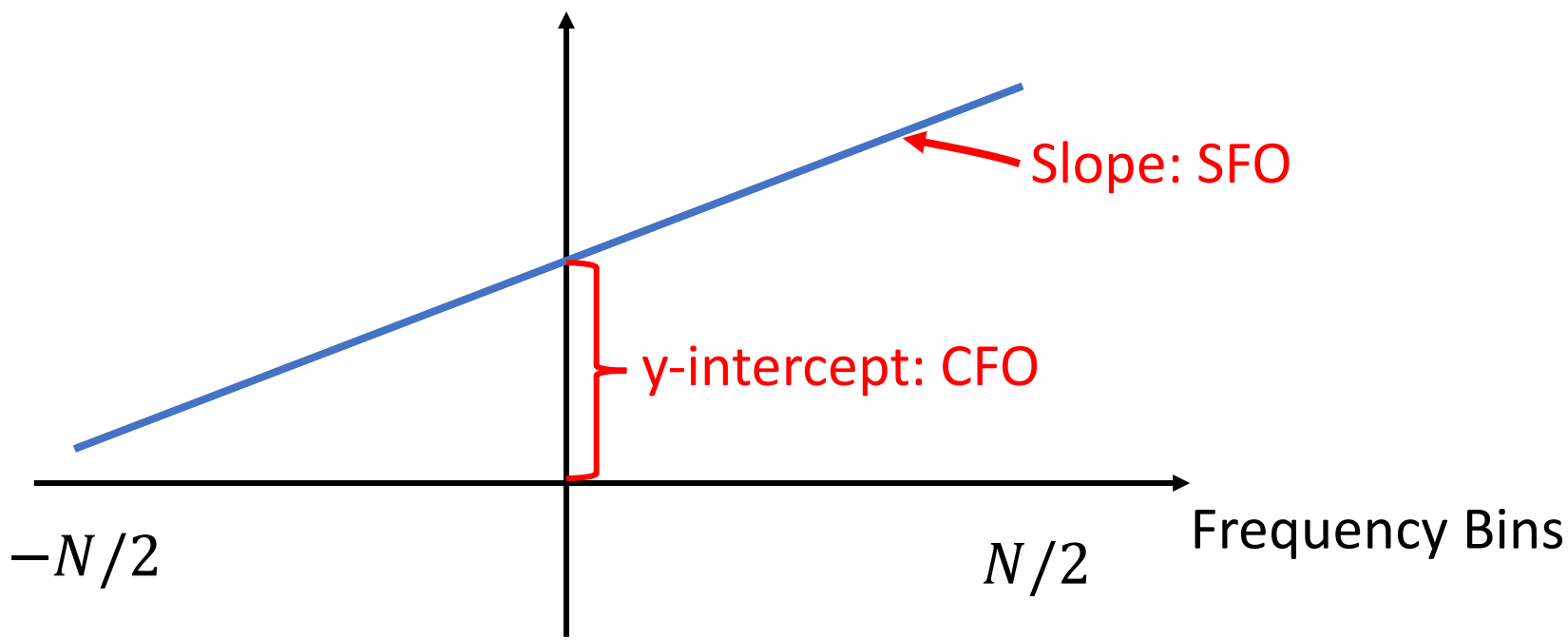
$$Y_2[f] = X_2[f]e^{j\frac{2\pi f (n+N+CP)\delta T_s}{N} - 2\pi\delta f_c (n+N+CP) T_s}$$

Phase accumulation: $\Delta\phi = 2\pi f \frac{(N+CP)\delta T_s}{N} - 2\pi\delta f_c (N+CP) T_s$

Phase Tracking

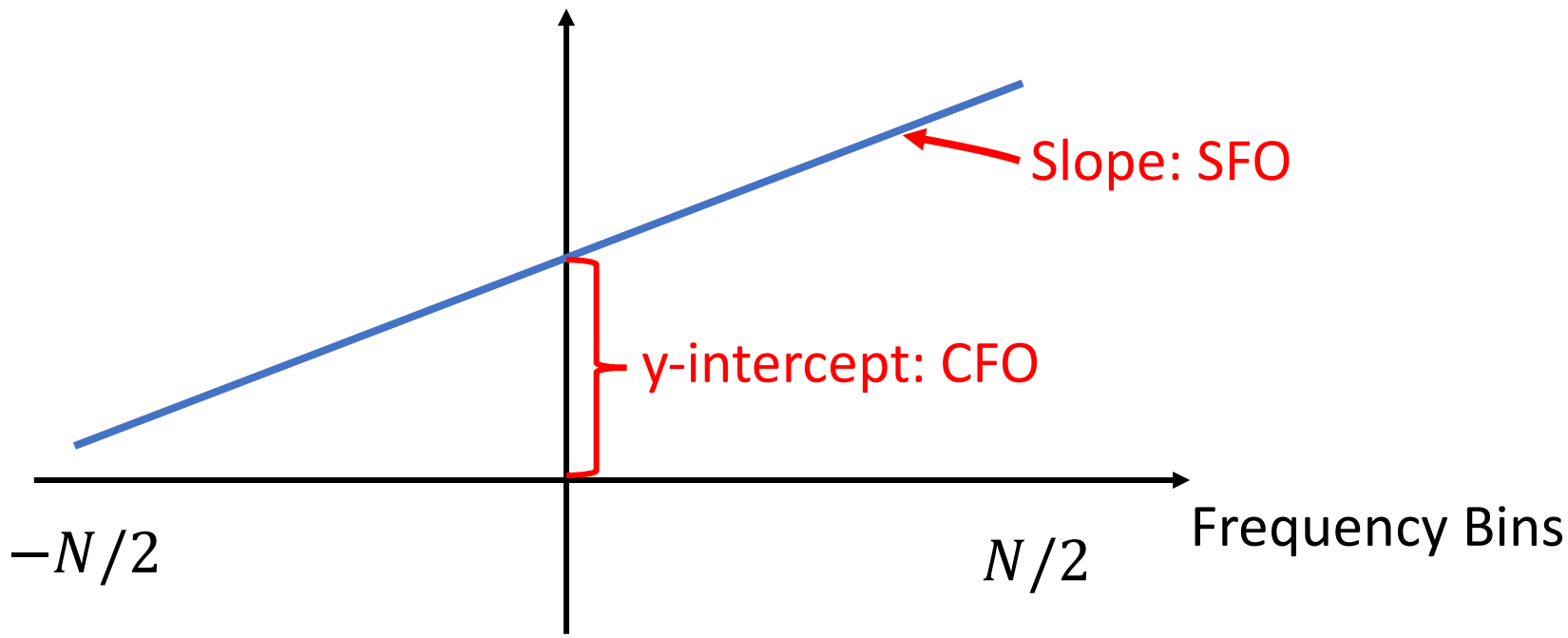
- Residual CFO (Carrier Frequency Offset)
- Residual SFO (Sampling Frequency Offset)

- Phase accumulation: $\Delta\phi = 2\pi f \frac{(N+CP)\delta T_s}{N} - 2\pi\delta f_c (N+CP) T_s$



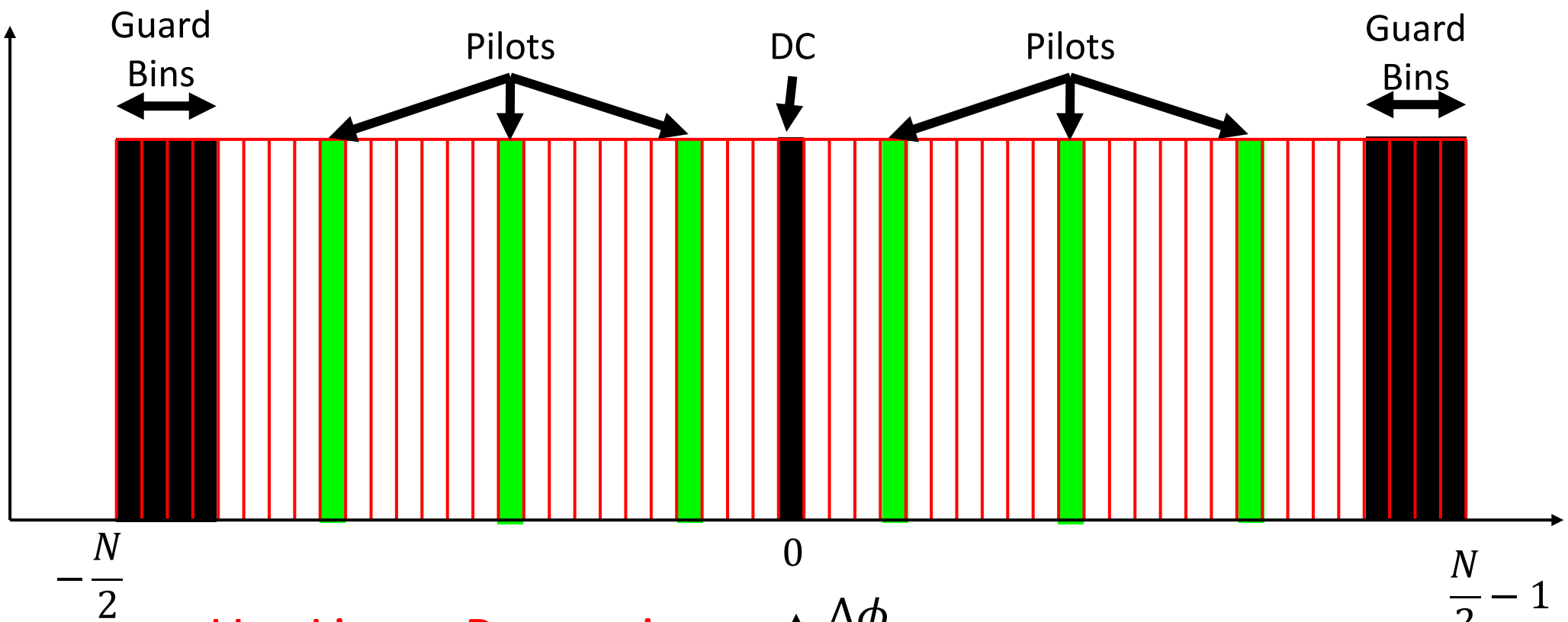
OFDM Phase Tracking

- Phase accumulation: $\Delta\phi = 2\pi f \frac{(N + CP)\delta T_s}{N} - 2\pi\delta f_c (N + CP) T_s$

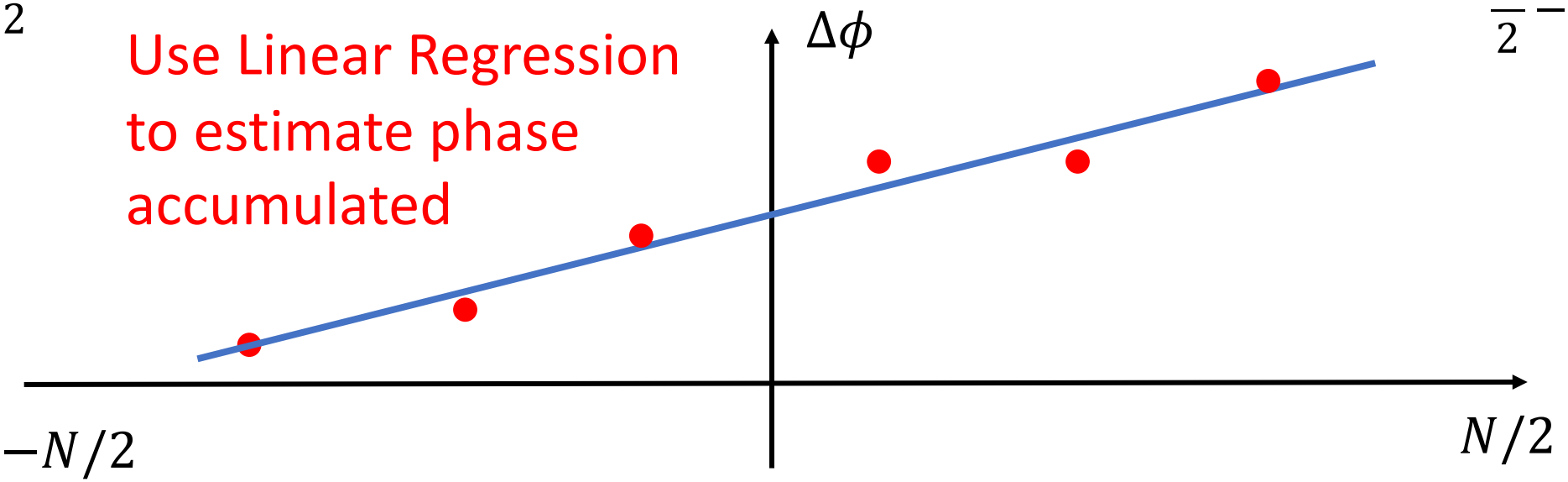


- Sufficient to estimate slope & y-intercept to know the phase accumulated for all subcarriers.
- Use only few subcarriers as pilots & send known bits in them.

OFDM Symbol



Use Linear Regression to estimate phase accumulated



OFDM: Putting it Together

At TX:

- Create preamble symbol from training sequence (Uses BPSK)

- Repeat preamble symbol:
  - 4 times for packet detection
  - 2 times for CFO estimation
  - 2 times for channel estimation
  - Add CP for the last preamble

- Create data symbol from:
  - Data bits (Uses BPSK, QPSK, M-QAM)
  - Pilot bits (Uses BPSK)

- Add cyclic prefix to data symbols.

OFDM: Putting it Together

At RX:

- Detect beginning of packet.
- Estimate & correct for CFO.
- Jump $\approx 0.75\ CP$ samples into symbol to avoid ISI
- Estimate the channel.
- For each subsequent data symbol:
  - Remove CP
  - Take FFT of Size N
  - Correct for channel
  - Use linear regression to estimate residual CFO and SFO
  - Estimate accumulated phase $\Delta\phi(f)$ for each frequency bin
  - Add $\Delta\phi(f)$ to channel estimate $\widetilde{H}(f)$
  - Decode Bits

Progress so far?

A. I understand and I can follow

B. I understand most stuff but not everything

C. I can follow but I do not understand everything

D. I cannot follow and understood nothing about OFDM