Wireless Communications
From 5G and WiFi 6 to Low Power IoT

Lecture 4: OFDM
Haitham Hassanieh
Yesterday’s Lecture Was

A. Too Difficult: I understood nothing.

B. Difficult: I understood something but not everything.

C. Moderate: I understood most stuff but I have questions

D. Easy: I understood everything really well.
Multipath in the Wireless Channel is problematic since it creates:

A. Inter-Symbol-Interference

B. Pathloss

C. Frequency Selective Fading

D. Additive White Gaussian Noise
ISI is considered negligible if the delayed symbols arriving longer paths interfere with less than < 1% of the symbol length.

If the longest path in the channel delays the symbol by 10ns, what is the maximum bandwidth for which we can ignore ISI?

A. 1 MHz
B. 10 MHz
C. 100 MHz
D. You can never ignore ISI
Previous Lecture:

- Pulse Shaping
- Matched Filter
- Multipath Channel
- Channel Estimation & Correction
- Narrowband vs. Wideband Channels
- Channel Equalization

This Lecture:

- Multi-Carrier Modulation
- Orthogonal Frequency Division Multiplexing (OFDM)
- OFDM Time Synchronization
- OFDM Frequency Synchronization
- OFDM Channel Estimation & Correction
- OFDM Phase Tracking
Single Carrier Modulation

Symbols modulated on a single carrier frequency

\[ s[n] \cos(2\pi f_c t) \]
Single Carrier Modulation

Symbols modulated on a single carrier frequency

- Low Spectral Efficiency: sinc & raised cosine leakage
- ISI: Inter-Symbol-Interference limits performance
Multi-Carrier Modulation

Symbols modulated on multiple Sub-carrier frequencies

- Divide spectrum into many narrow bands

\[ x(t) = \sum_{i} s_i[n]e^{-j2\pi f_i t} \]

- Transmit symbols on different carriers in narrow bands
- Channel is Flat \( \rightarrow \) No need to worry about ISI
Multi-Carrier Modulation

Symbols modulated on multiple Sub-carrier frequencies

- Divide spectrum into many narrow bands

\[ x(t) = \sum_i s_i[n]e^{-j2\pi f_it} \]

\[ y(t) = \sum_i h_i s_i[n]e^{-j2\pi f_it} \]

- Transmit symbols on different carriers in narrow bands

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Multi-Carrier Modulation

Symbols modulated on multiple Sub-carrier frequencies

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\[ y(t) = \sum_{i} h_i s_i[n] e^{-j2\pi f_i t} \]

• Transmit symbols on different carriers in narrow bands

• Channel is Flat → No need to worry about ISI

Not That Simple!
Multi-Carrier Modulation

Symbols modulated on multiple Sub-carrier frequencies

- Divide spectrum into many narrow bands
- Significant Leakage between adjacent subcarriers
- Need Guard Bands $\rightarrow$ Very inefficient!

Solution: Make the Sub-Carriers Orthogonal
Multi-Carrier Modulation

Symbols modulated on multiple Sub-carrier frequencies

Make the Sub-Carriers Orthogonal
OFDM: Orthogonal Frequency Division Multiplexing

- Subcarriers are orthogonal: At the sub-carrier frequency, the sampled value has zero leakage from other subcarriers.

- Subcarrier separation can be very small, for N subcarriers and bandwidth B:

\[ \Delta f = \frac{B}{N} \]
• Subcarriers are orthogonal: At the sub-carrier frequency, the sampled value has zero leakage from other subcarriers.

• Subcarrier separation can be very small, for N subcarriers and bandwidth B:

$$\Delta f = \frac{B}{N}$$

How to Achieve This?
OFDM: Orthogonal Frequency Division Multiplexing

Use DFT: Discrete Fourier Transform

N-Point DFT: \( X(f_i) = \frac{1}{N} \sum_{t=0}^{N-1} x(t)e^{-j\frac{2\pi f_i t}{N}} \)

N-Point IDFT: \( x(t) = \sum_{f_i=0}^{N-1} X(f_i)e^{j\frac{2\pi f_i t}{N}} \)

Send symbols in Frequency Domain

\( X(f_i) = s[n] \rightarrow \) Compute and transmit \( x(t) \) using IDFT
OFDM: Orthogonal Frequency Division Multiplexing

Send symbols in Frequency Domain

\[ X(f_i) = s[n] \rightarrow \text{Compute and transmit } x(t) \text{ using IDFT} \]

- \( N \) subcarrier \( \rightarrow \) IDFT of length \( N \)
- Symbols \( s[n] \) can come from any modulation: BPSK, QPSK, QAM...
- \( x(t) \) is complex \( \rightarrow \) need \( I \) & \( Q \) \( \rightarrow \) No point using PAM or ASK ...
- OFDM Symbol: \( N \) samples of \( x(t) \) generated from the same modulated symbols using IDFT.
- OFDM Symbol Time: \( T = N/B \) where \( B \) is the bandwidth.
- OFDM Frequency Bin Width: \( \Delta f = 1/T = B/N \)
OFDM: Orthogonal Frequency Division Multiplexing

Transmitter

Bits $\rightarrow$ Modulation $\rightarrow$ Serial to Parallel $\rightarrow$ IFFT $\rightarrow$ Parallel to Serial $\rightarrow$ Demodulation $\rightarrow$ Bits

Receiver

Bits $\rightarrow$ LNA $\rightarrow$ BPF $\rightarrow$ MIXER $\rightarrow$ LPF $\rightarrow$ ADC $\rightarrow$ I $\rightarrow$ Serial to Parallel $\rightarrow$ FFT $\rightarrow$ Parallel to Serial $\rightarrow$ Demodulation $\rightarrow$ Bits

Symbols:
- $I$: In-phase component
- $Q$: Quadrature component
- PA: Power Amplifier
• FFT can be represented 0 to $N - 1$ or $N/2$ to $N/2 - 1$.
• OFDM Symbol created in digital baseband $\rightarrow$ 0 bin corresponds to DC

$$X(0) = \frac{1}{N} \sum_{t=0}^{N-1} x(t) e^{-j \frac{2\pi 0 t}{N}} = \frac{1}{N} \sum_{t=0}^{N-1} x(t) = DC$$
OFDM Symbol in Frequency Domain

- FFT can be represented 0 to \( N - 1 \) or \( N/2 \) to \( N/2 - 1 \).
- OFDM Symbol created in digital baseband \( \rightarrow \) 0 bin corresponds to DC.
- DC of the circuits corrupts bits sent on the 0 bin \( \rightarrow \) Do not use 0 bin.
• FFT can be represented 0 to \( N - 1 \) or \( N/2 \) to \( N/2 - 1 \).

• OFDM Symbol created in digital baseband \( \rightarrow \) 0 bin corresponds to DC

• DC of the circuits corrupts bits sent on the 0 bin \( \rightarrow \) Do not use 0 bin
- Subcarriers orthogonal to each other but not to near by channels.
- Need Guard Bins at sides of the channel \(\rightarrow\) Transmit nothing there
• Subcarriers orthogonal to each other but not to near by channels.

• Need Guard Bins at sides of the channel $\rightarrow$ Transmit nothing there

• Reduce Number of Guard band from $N$ to 2 $\rightarrow$ Very Spectrally Efficient
OFDM: Orthogonal Frequency Division Multiplexing

Transmit Symbols in Frequency Domain On Orthogonal Subcarriers

**TX**

- Bits ▸ Modulation ▸ Serial to Parallel ▸ IFFT ▸ Parallel to Serial ▸ Re{ } ▸ DAC ▸ LPF ▸ PLL ▸ Mixer ▸ BPF ▸ PA

**RX**

- LNA ▸ BPF ▸ PLL ▸ Mixer ▸ 90° ▸ BPF ▸ Mixer ▸ LPF ▸ ADC ▸ I ▸ + ▸ jQ ▸ IFFT ▸ Parallel to Serial ▸ Demodulation ▸ Bits
OFDM Symbol

Bits: 1 0 1 0 1 0 0 0 1 1 0 1 1 1 0 0

..., +1, −1, +1, −1, +1, −1, −1, +1, +1, −1, +1, +1, −1, −1, ...

Guard Bins

DC

Guard Bins

\(-\frac{N}{2}\)

0

\(\frac{N}{2} - 1\)

IFFT

Symbol in Time
Not That Simple
OFDM Symbol

... FFT Window ...

... S1 S2 S3 S4 ...

FFT

..., +1, −1, +1, ...

... 101 ...
OFDM Symbol

\[\cdots\quad S1 \quad S2 \quad S3 \quad S4 \quad \cdots\]

FFT Window

\[
\begin{align*}
\text{FFT} & \quad \downarrow \quad \text{FFT}
\end{align*}
\]

\[
\begin{align*}
\cdots, +1, -1, +1, \cdots & \quad \downarrow \quad \cdots, +1, +1, +1, \cdots \\
\text{... 101 ...} & \quad \downarrow \quad \text{... 111 ...}
\end{align*}
\]
OFDM Symbol

\[ \ldots \]

\begin{align*}
S1 & & S2 & & S3 & & S4 & & \ldots \\
\text{FFT} & & \text{FFT} & & \text{FFT} & & \text{FFT Window} & & \ldots \\
\ldots, +1, -1, +1, \ldots & & \ldots, +1, +1, +1, \ldots & & \ldots, -1, -1, +1, \ldots & & \ldots \\
\downarrow & & \downarrow & & \downarrow & & \downarrow & & \ldots \\
\ldots 101 \ldots & & \ldots 111 \ldots & & \ldots 001 \ldots & & \ldots \\
\end{align*}
OFDM Symbol

![Diagram of OFDM symbol with FFT windows and sequences]

Assumes FFT window is perfectly aligned with symbol boundaries
OFDM Symbol

... S1 S2 S3 S4 ...

FFT Window

FFT

..., +0.5 + 1i, −0.7 + 0.3i, ...

× Cannot decode!

FFT window is misaligned with symbol

Subcarriers are no longer orthogonal.
OFDM Cyclic Prefix

- DFT (FFT) assumes time samples are periodic of period $N$

- Circular Shift before taking FFT:

$$x[t] \rightarrow X[f]$$

$$x[t - \tau \mod N] \rightarrow X[f]e^{-j\frac{2\pi f \tau}{N}}$$
• DFT (FFT) assumes time samples are periodic of period $N$

• Circular Shift before taking FFT:

$$x[t] \rightarrow X[f]$$

$$x[t - \tau \mod N] \rightarrow X[f]e^{-j\frac{2\pi f \tau}{N}}$$
• Even if FFT window is misaligned, CP ensures that all samples come from the same symbol $\Rightarrow$ Orthogonality is preserved!

• Cyclic Prefix can be created by:
  o Take first few samples and append them to end of symbol.
  o Take last few samples and prefix them to beginning of symbol.

• Simple Phase Shift $\Rightarrow$ Can be corrected by lumping with channel $H[f]$
OFDM Cyclic Prefix

Cyclic Prefix:

- Preserves orthogonality by allowing some misalignment in FFT Window
- Deals with Inter-Symbol-Interference
OFDM Cyclic Prefix

Cyclic Prefix:

- Preserves orthogonality by allowing some misalignment in FFT Window
- Deals with Inter-Symbol-Interference

![Diagram showing cyclic prefix impact on FFT Window with and without ISI](image)
OFDM Cyclic Prefix

Cyclic Prefix:

• Preserves orthogonality by allowing some misalignment in FFT Window

• Deals with Inter-Symbol-Interference

![Diagram showing cyclic prefix with FFT Windows and no ISI in the windows]
OFDM Cyclic Prefix

Cyclic Prefix:

+ Preserves orthogonality by allowing some misalignment in FFT Window
+ Deals with Inter-Symbol-Interference

Overhead: Send $CP + N$ samples for every $N$ samples

$$\text{Overhead} = \frac{CP}{CP + N}$$

e.g. WiFi 802.11n: $N = 64, CP = 16 \rightarrow \text{Overhead} = 20\%$

e.g. LTE: $N = 1024, CP = 72 \rightarrow \text{Overhead} = 6.5\%$
Progress so far?

A. I understand and I can follow
B. I understand most stuff but not everything
C. I can follow but I do not understand everything
D. I cannot follow and understood nothing about OFDM
Cyclic prefix is not a bulletproof solution.

Can still end up misaligned!

Need a way to ensure we detect the beginning of the packet correctly.

If we do, CP will ensure that even if we are not accurate, we can still decode.
OFDM Packet Detection

• Detect Beginning of packet to make sure we are within the CP

• Send Training Sequence: Preamble Symbols

• Preamble Symbols: Known Symbol Repeated at the beginning of packet

  Preamble Preamble ⋯ Preamble S1 CP1 S2 CP2 ⋯

• No need for CP with preamble symbols
OFDM Packet Detection: Sliding Window

- Two windows of $L$ ($2N$) samples each.

- Compute:
  \[
  \frac{P_B}{P_A} = \frac{\sum_{k=t}^{t+2L} |x[k]|^2}{\sum_{k=t}^{t+L} |x[k]|^2}
  \]
OFDM Packet Detection: Sliding Window

- Two windows of $L \times (2N)$ samples each.

- Compute: \[
\frac{P_B}{P_A} = \frac{\sum_{k=t+L}^{t+2L} |x[k]|^2}{\sum_{k=t}^{t+L} |x[k]|^2}
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OFDM: Orthogonal Frequency Division Multiplexing

Transmit Symbols in Frequency Domain On Orthogonal Subcarriers

So far, we assumed carriers generated by LOs are perfectly synchronized!
Carrier Frequency Offset

Assumes TX & RX perfectly synched
Carrier Frequency Offset

\[ x(t) \rightarrow x(t) e^{-j2\pi f_c t} \rightarrow h(t) \ast x(t) e^{-j2\pi f_c t} \rightarrow h(t) \ast x(t) e^{-j2\pi f_c t} \times e^{j2\pi f'_c t} \]

TX & RX are not synched

CFO: \( \Delta f_c = f_c - f'_c \)

\[ y(t) = h(t) \ast x(t) e^{-j2\pi \Delta f_c t} + v(t) \]

Phase changes with time!
Carrier Frequency Offset

Consider BPSK Modulation.

\[ 0 \rightarrow -1 \]
\[ 1 \rightarrow +1 \]

\[ x(t) \quad h x(t - \tau) e^{-j2\pi\Delta f_c t} + v(t) \]
Carrier Frequency Offset

Consider BPSK Modulation.

\[ 0 \rightarrow -1 \]
\[ 1 \rightarrow +1 \]

Impossible to Decode!
Carrier Frequency Offset
Consider 16 QAM Modulation

Need to estimate and correct CFO to decode!
OFDM CFO Estimation & Correction

• Use Preamble to estimate CFO

\[ y_1(t) = x(t)e^{-j2\pi \Delta f_c t} \]

\[ y_2(t) = x(t)e^{-j2\pi \Delta f_c (t+NT_s)} \]
OFDM CFO Estimation & Correction

- Use Preamble to estimate CFO

\[ y_1[n] = x[n]e^{-j2\pi\Delta f_c nT_s} \]

\[ y_2[n] = x[n]e^{-j2\pi\Delta f_c (nT_s + NT_s)} \]

- Compute: \[ A = \sum_{t=1}^{N} y_1[n]y_2^*[n] = \sum_{t=1}^{N} x[n]x^*[n]e^{j2\pi\Delta f_c NT_s} \]

\[ = e^{j2\pi\Delta f_c NT_s} \sum_{t=1}^{N} |x[n]|^2 \rightarrow \Delta f_c = \frac{\angle A}{2\pi NT_s} \]
OFDM CFO Estimation & Correction

- Use Preamble to estimate CFO

\[ y_1[n] = x[n]e^{-j2\pi \Delta f_c n T_s} \]
\[ y_2[n] = x[n]e^{-j2\pi \Delta f_c (n T_s + N T_s)} \]

- Compute: \( A = \sum_{t=1}^{N} y_1[n] y_2^*[n] \Rightarrow \Delta f_c = \frac{\angle A}{2\pi N T_s} \)

- Correct CFO: \( y[n] \times e^{j2\pi \Delta f_c n T_s} \)
We use the following equation to estimate CFO: \( \Delta f_c = \frac{\angle A}{2\pi NT_s} \).

Suppose \( f_c = 5 \text{ GHz} \), the bandwidth = 10 MHz and the clock precision is 20ppm. For what values of N will the above equation estimate the CFO incorrectly?

A. N < 10

- Equation give wrong result when the phase of A wraps around 2\( \pi \)! We need: \( |\angle A| \leq \pi \)

B. N < 20

- \( \Delta f_c = 5 \text{ GHz} \times \frac{20}{1000000} = 100 \text{ kHz} \)

C. N > 50

- \( T_s = \frac{1}{10\text{MHz}} = 0.1\mu s \)

D. N < 50

\[
\Delta f_c \leq \frac{1}{2NT_s}
\]
OFDM Channel Estimation

- Use Preamble to estimate the channel

\[ y(t) = h(t) * x(t) \leftrightarrow Y(f) = H(f) X(f) \]

- Send \( X(f) \): \(-1, +1, -1, -1, -1, +1, \ldots\)

- Receive: \(-H(1), H(2), -H(3), -H(4), -H(5), H(6), \ldots\)

- Estimate: \( \tilde{H}(f) = \frac{Y(f)}{X(f)} \)

- Use two preambles to average noise: \( \tilde{H}(f) = \frac{Y_1(f) + Y_2(f)}{2 X(f)} \)
Phase Tracking

So Far: Estimated and Corrected For Coarse Value of CFO

- Residual CFO:

\[ y(t) = h(t) x(t) e^{-j2\pi \Delta f_c t} + v(t) \]

\[ \Delta f_c = d f_c + \delta f_c \]

Coarse CFO  Residual CFO

We estimated and corrected for coarse CFO!

Even small residual can accumulate over time to create large phase: \( e^{-j2\pi \delta f_c t} \)

Need to track the phase
Phase Tracking

- Residual CFO (Carrier Frequency Offset)
- Residual SFO (Sampling Frequency Offset)
Phase Tracking

- Residual CFO (Carrier Frequency Offset)
- Residual SFO (Sampling Frequency Offset)

When we sample the signal there is a residual sampling offset: \( n\delta T_s \)

\[
y[n] = x[n + n\delta T_s]e^{-j2\pi \delta f_c nT_s}
\]

\[
= \sum_{f=0}^{N-1} X[f]e^{j \frac{2\pi f (n+n\delta T_s)}{N}} e^{-j2\pi \delta f_c nT_s}
\]

\[
Y[f] = X[f]e^{j \frac{2\pi fn\delta T_s}{N} - 2\pi \delta f_c nT_s}
\]
Phase Tracking

• Residual CFO (Carrier Frequency Offset)

\[ Y_1[f] = X_1[f]e^{j\frac{2\pi f n \delta T_s}{N} - 2\pi \delta f_c n T_s} \]

\[ Y_2[f] = X_2[f]e^{j\frac{2\pi f (n+N+CP) \delta T_s}{N} - 2\pi \delta f_c (n+N+CP) T_s} \]

Phase accumulation: \[ \Delta \phi = 2\pi f \frac{(N + CP) \delta T_s}{N} - 2\pi \delta f_c (N + CP) T_s \]
Phase Tracking

- Residual CFO (Carrier Frequency Offset)
- Residual SFO (Sampling Frequency Offset)

Phase accumulation: \( \Delta \phi = 2\pi f \frac{(N + CP)\delta T_s}{N} - 2\pi \delta f_c (N + CP)T_s \)
OFDM Phase Tracking

- Phase accumulation: \( \Delta \phi = 2\pi f \frac{(N + CP)\delta T_s}{N} - 2\pi \delta f_c (N + CP)T_s \)

- Sufficient to estimate slope & y-intercept to know the phase accumulated for all subcarriers.

- Use only few subcarriers as pilots & send known bits in them.
Use Linear Regression to estimate phase accumulated
OFDM: Putting it Together

At TX:

- Create preamble symbol from training sequence (Uses BPSK)

- Repeat preamble symbol:
  - 4 times for packet detection
  - 2 times for CFO estimation
  - 2 times for channel estimation
  - Add CP for the last preamble

- Create data symbol from:
  - Data bits (Uses BPSK, QPSK, M-QAM)
  - Pilot bits (Uses BPSK)

- Add cyclic prefix to data symbols.
OFDM: Putting it Together

At RX:

- Detect beginning of packet.
- Estimate & correct for CFO.
- Jump $\approx 0.75 \ CP$ samples into symbol to avoid ISI
- Estimate the channel.
- For each subsequent data symbol:
  - Remove CP
  - Take FFT of Size N
  - Correct for channel
  - Use linear regression to estimate residual CFO and SFO
  - Estimate accumulated phase $\Delta \phi (f)$ for each frequency bin
  - Add $\Delta \phi (f)$ to channel estimate $\tilde{H}(f)$
  - Decode Bits
Progress so far?

A. I understand and I can follow

B. I understand most stuff but not everything

C. I can follow but I do not understand everything

D. I cannot follow and understood nothing about OFDM